

csci 210: Data Structures
More Recursion

Summary

- Topics: more recursion
 - Subset sum: finding if there exists a subset of an array that sum up to a given target
 - Permute: finding all permutations of a given string
 - Subset: finding all subsets of a given string

Thinking recursively

- Finding the recursive structure of the problem is the hard part
- Common patterns
 - divide in half, solve one half
 - divide in sub-problems, solve each sub-problem recursively, “merge”
 - solve one or several problems of size $n-1$
 - process first element, recurse on remaining problem
- Recursion
 - functional: function computes and returns result.
 - Example: computing the sum of n numbers; isPalindrome; binary search.
 - procedural: no return result (function returns void). The task is accomplished during the recursive calls.
 - Example: Sierpinski fractals.
- Recursion
 - exhaustive
 - non-exhaustive: stops early

Subset Sum

- Given an array of numbers and a target value, find whether there exists a subset of those numbers that sum up to the target value.

//return true if there exists such a subset, and false otherwise

boolean subsetSum (int[] a, int target)

- Example:
- Recursive structure:
 - consider the next element in the array
 - try making a sum WITH this element
 - try making a sum WITHOUT this element
 - if neither is possible, return false

Subset Sum

- So: consider the next element, it is either in the solution, or not. Try both ways. If both fail, return false.
- Need to keep track of the partial sum so far. When starting a recursive call, need to know the sum of the current subset. Also need to know the index of the next element to consider.

```
void recSubset(int[] a, int target, int i, int sumSoFar)
```

- The problem asked for a subsetSum function with the following signature:

```
boolean subsetSum (int[] a, int target)
```

- Need a wrapper:

```
boolean subsetSum (int[] a, int target) {  
    return recSubset(a, target, 0, 0);  
}
```

Subset Sum

```
//i is the index of the next element to consider
//sumSoFar is the sum of elements included in the solution so far.
boolean recSubset(int[] a, int target, int i, int sumSoFar) {
    //basecases
    //we got it
    if (sumSoFar == target) return true;
    //we reached the end and sum is not equal to target
    if (i == a.length) return false;

    //recursive case: try next element both in and out of the sum
    boolean with = recSubset(a, target, i+1, sumSoFar + a[i]);
    boolean without = recSubset(a, target, i+1, sumSoFar);
    return (with || without);
}
```

Subset Sum

- The tree of recursive calls for `recSubset([1, 2, 3, 4], target, 0, 0)`

Subset Sum

- You may notice that there is no need to keep both target and sumSoFar as arguments in recSubset; instead, use target, and subtract from it when you include an element in a set.
- //i is the index of the next element to consider

```
boolean recSubset(int[] a, int target, int i, int sumSoFar) {  
    //basecases  
    //we got it  
    if (target==0) return true;  
    //we reached the end and sum is not equal to target  
    if (i == a.length) return false;  
  
    //recursive case: try next element both in and out of the sum  
    boolean with = recSubset(a, target-a[i], i+1);  
    boolean without = recSubset(a, target, i+1);  
    return (with || without);  
}
```


Subset Sum

- Variations
 - Alternative strategy: at each step, chose one of the remaining element to be part of the subset and recurse on the remaining part.
 - How could you change the function so that it prints the elements of the subset that sum to target?
 - store partial subsets in another array
 - or print element at the end of recursive call
 - How could you change the function to report not only if such a subset exists, but to count all such subsets?

Permutations

- Write a function to print all permutations of a given string.
- Example: permute “abc” should print: abc, acb, bca, bac, cab, cba.

```
void printPerm(String s)
```

- Recursive structure:
 - Chose a letter from the input, and make this the first letter of the output
 - Recursively permute remaining input
 - chose a, permute “bc”: should generate “a” + all permutations of “bc”
 - chose all letters in turn to be first letters
 - chose b, permute “ac”: should generate “b” + all permutations of “ac”
 - chose c, permute “ab”: should generate “c” + all permutations of “ab”

- What is the base case?
- Can you make sure that each permutation is generated precisely once?

Permutations

- So: pick a letter, add it to the solution, recurse on remaining
- When starting a recursive call, we know the list of letters chosen so far; that is, we know the first part of the permutation generated so far.
- Need to keep track of it.

```
//print soFar + all permutations of remaining string  
void recPermute(String soFar, String remaining)
```

- The problem asked for a printPermute with a different signature: we need a wrapper

```
//print all permutations of s  
void printPerm (String s) {  
    recPermute("", s);  
}
```

- Why use wrappers? the user does not need to know the internals of the implementation. In this case, that it is recursive.

Permutations

```
//prints soFar+all permutations of remaining
```

```
void recPermute(String soFar, String remaining) {
```

```
    //base case
```

```
    if (remaining.length() == 0)
```

```
        System.out.println(soFar);
```

```
    else {
```

```
        for (int i=0; i< remaining.length(); i++) {
```

```
            String nextSoFar = soFar + remaining[i];
```

```
            String nextRemaining = remaining.substring(0,i) + remaining.substring(i+1);
```

```
            recPermute(nextSoFar, nextRemaining)
```

```
        }
```

```
    }
```

```
}
```

Permutations

- The tree of recursive calls for `recPermute("", "abc")`

Subsets

- Enumerate all subsets of a given string
- Example: subsets of “abc” are a, b, c, ab, ac, bc, abc
 - Order does not matter: “ab” is the same as “ba”
- Recursive structure
 - chose one element from input
 - can either include it in current subset or not
 - recursively form subsets including it
 - recursively form subsets excluding it
 - make sure to generate each set once
 - base case?

Subsets

```
void recSubsets(String soFar, String remaining) {  
    if (remaining.length()==0)  
        System.out.println(soFar);  
    else {  
        //add to subset, remove from rest, recurse  
        recSubsets(soFar+remaining[0], remaining.substring(1));  
        //don't add to subset, remove from rest, recurse  
        recSubsets(soFar, remaining.substring(1));  
    }  
}
```

```
void subsets(String s) {  
    recSubsets("", s);  
}
```

Subsets

- The tree of recursive calls for `recSubsets("", "abcd")`