csci 210: Data Structures

Stacks and Queues in Solution Searching

Summary

- Topics
  - Using Stacks and Queues in searching
  - Applications:
    - In-class problem: missionary and cannibals
    - In-class problem: finding way out of a maze
  - Searching a solution space: Depth-first and breadth-first search (DFS, BFS)

- READING:
  - GT textbook chapter 5

Searching in a Solution Space

- Remember the problems:
- Permutations: Write a function to print all permutations of a given string.
- Subsets: Write a function to enumerate all subsets of a given string
- Subset sum: Given an array of numbers and a target value, find whether there exists a subset of those numbers that sum up to the target value.

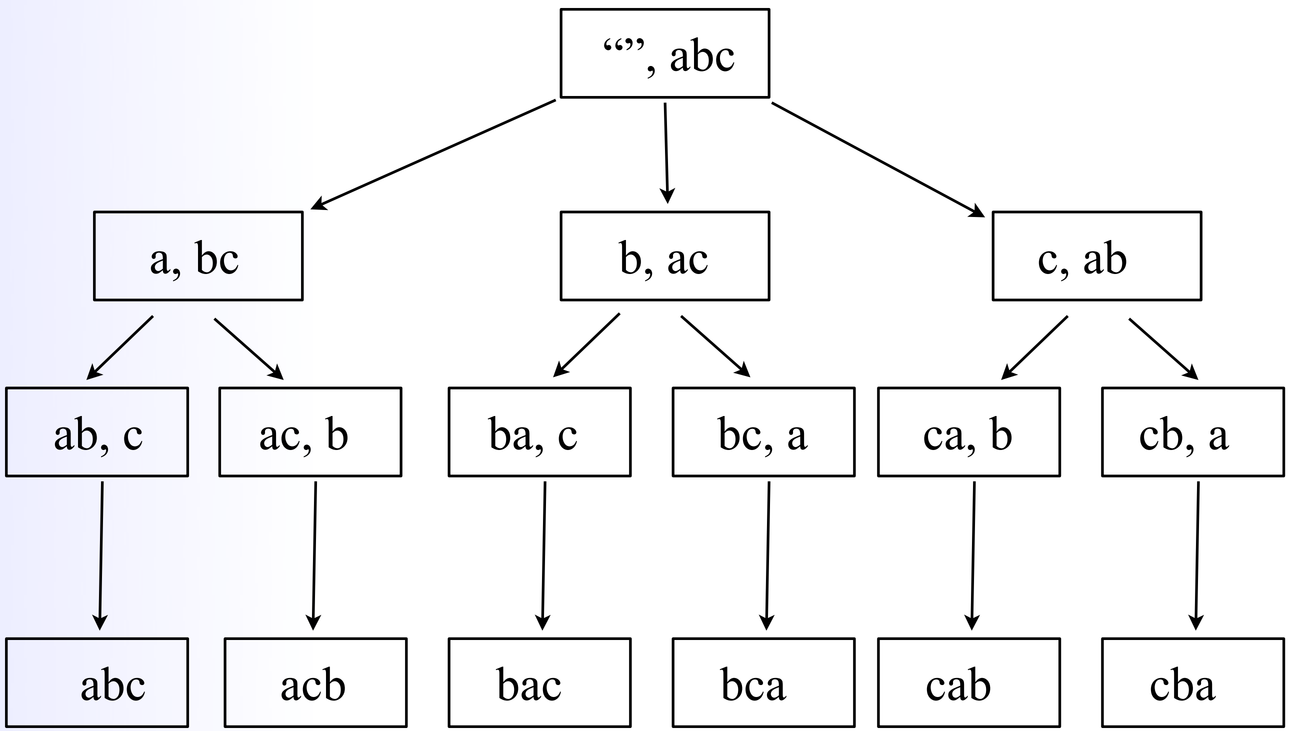
- We saw how to solve them recursively.
  - Idea: A recursive solution takes as parameters the partial solution so far. Given this partial solution, it finds all possible ways to build new solutions.

Recursive Permute

```
void  recPermute(String soFar, String remaining) {

   //base case
   if (remaining.length() == 0)
      System.out.println(soFar);
   else {
      for (int i=0; i< remaining.length(); i++) {
         String nextSoFar = soFar + remaining[i];
         String nextRemaining = remaining.substring(0,i) +
         remaining.substring(i+1);
         recPermute(nextSoFar, nextRemaining)
      }
   }
}
```

Tree of recursive calls



Searching in a Solution Space

- Permutations: Write a function to print all permutations of a given string.
- Subsets: Write a function to enumerate all subsets of a given string
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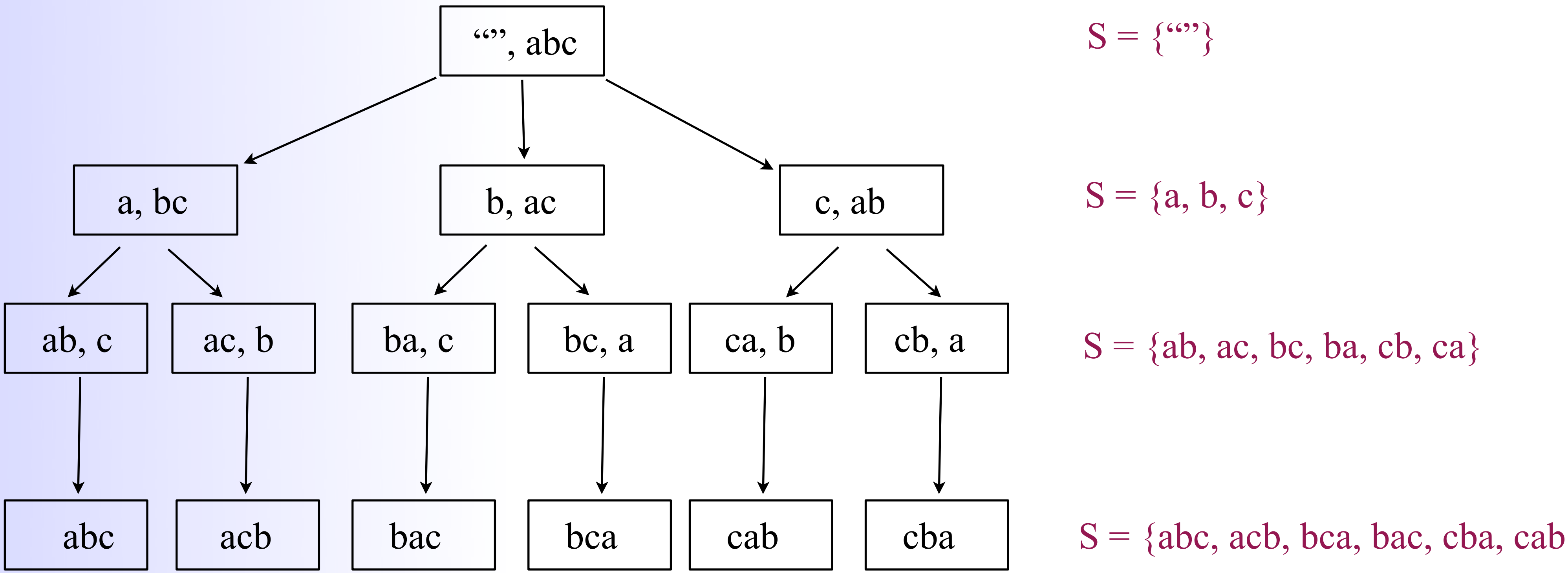
- We saw how to solve them recursively.
  - Idea: A recursive solution takes as parameters the partial solution so far. Given this partial solution, it finds all possible ways to build new solutions.

- Another way to look at it:
  - let S = the set of all possible partial solutions so far.
    - e.g. S = {a, b, c, d } //all possible partial solutions of one letter
  - for each partial solution p in S
    - move one step forward and find all possible next solutions from p. Add all these to a new set S'.
    - e.g. partial solution p = "a" gives 3 new solutions: "ab", "ac" , "ad"
  - repeat with S = S'
    - e.g. S' = {ab, ac, ad, ba, bc, bd, ca, cb, cd, da, db, dc}

Permutations

- Recursive permute:
  - recPermute(soFar, remaining)
  - the function knows about the "current" partial solution
  - the system keeps track of the active calls---the tree of recursive calls corresponds to all partial solutions



- Non-recursive permute
  - construct explicitly the set of partial solutions

Building a Solution

- Imagine that we encode the partial solution to a problem in some way
  - for e.g. for permutations a partial solution could be a tuple s = <soFar, remaining>

- //S denotes the set of partial solutions
- S = empty set
- //create the initial state
- S = { initial-state}
- while S is not empty
  - S' = {}
  - go through all partial solution s from S
  - for each s generate all possible next solutions from s and add them to S'
  - S = S'

- Think of S as the (partial) solution space. Our algorithm will construct it.

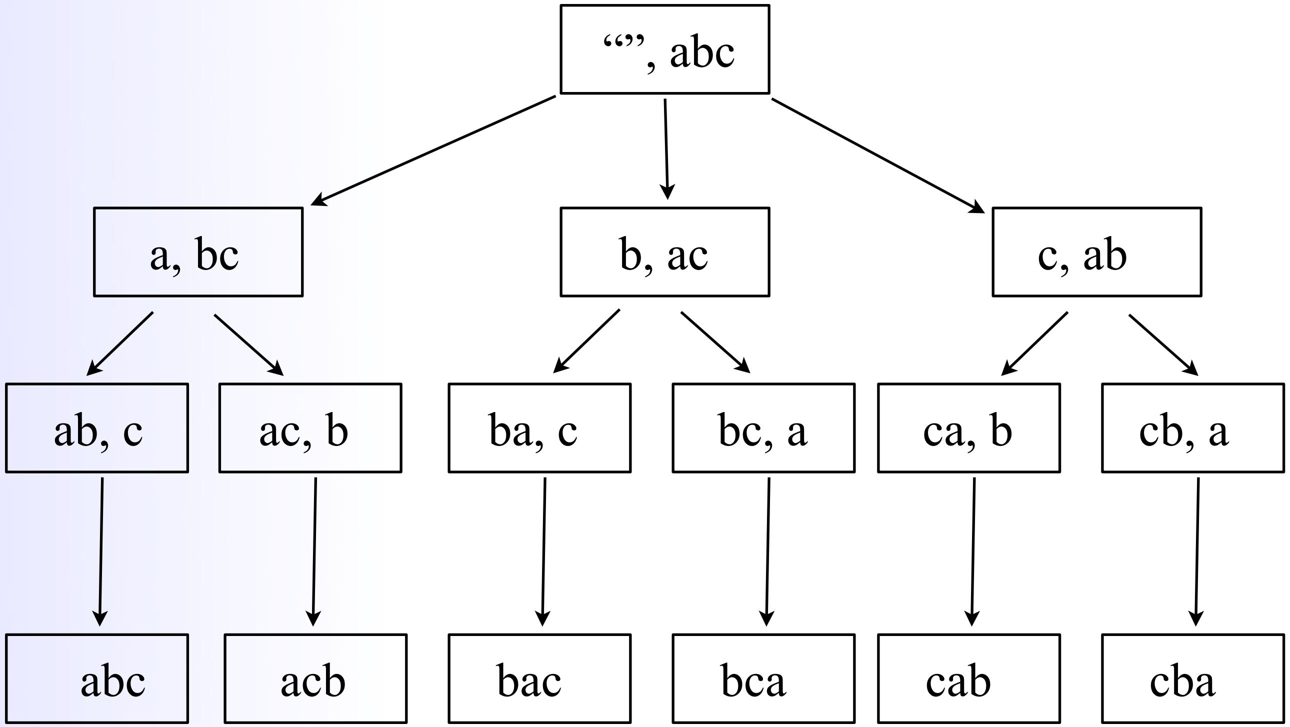
Building a Solution

- We do not need both S and S'
- Think of S as the (partial) solution space. Our algorithm will construct it.

- `S = empty set`
- `//create the initial state`
- `S = { initial-state}`
- `while S is not empty`
  - `delete the next partial solution s from  S`
  - `generate all possible next solutions from s and add them to S`

The solution space

- Each solution is a state
- Each solution generates new solutions



Building a Solution

- Think of S as the solution space. Our algorithm will construct it.

- `S = empty set`
- `//create the initial state`
- `S = { initial-state}`
- `while S is not empty`
  - `delete the next partial solution s from  S`
  - `generate all possible next solutions from s and add them to S`

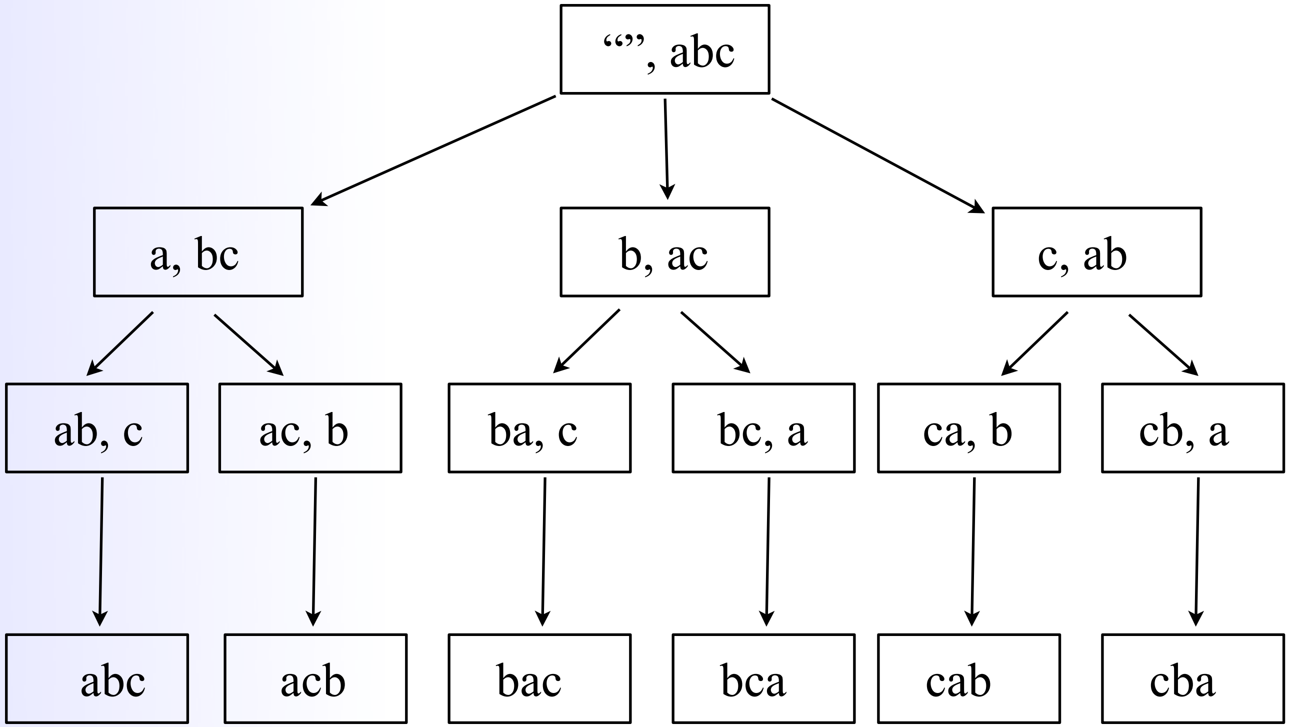
- S is a set of states. How to store S ?
- Keep S as a queue
  - delete next solution from the front
  - add new solutions to the end of queue
- Keep S as a stack
  - delete next solution from the top
  - add new solutions to the top

- S = empty set
- //create the initial state
- S = { initial-state}
- while S is not empty
  - delete the next partial solution s from S
  - generate all possible next solutions from s and add them to S

- S as a queue
  - S = { <"", "abc">}
  - partial solution s = <"", abc> generates 3 new solutions <a, bc>, <b, ac>, <c, ab>
  - they are all put in S: S = {<a, bc>, <b, ac>, <c, ab>}
  - partial solution s=<a,bc> generates 2 new solutions <ab,c> and <ac,b>; they are put in S
  - S = { <b, ac>, <c, ab>, <ab,c>, <ac,b>}
  - S = { <c, ab>, <ab,c>, <ac,b>, <ba,c>, <bc, a>}
  - S = { <ab,c>, <ac,b>, <ba,c>, <bc, a>, <ca,b>, <cb, a>}
  - …
  - S = { <abc,"">, <acb,"">, <bac,"">, <bca,"">, <cab,"">, <cba,"">}
  - S = {}

The solution space

- How does the algorithm traverse and construct the solution space when S is a queue?



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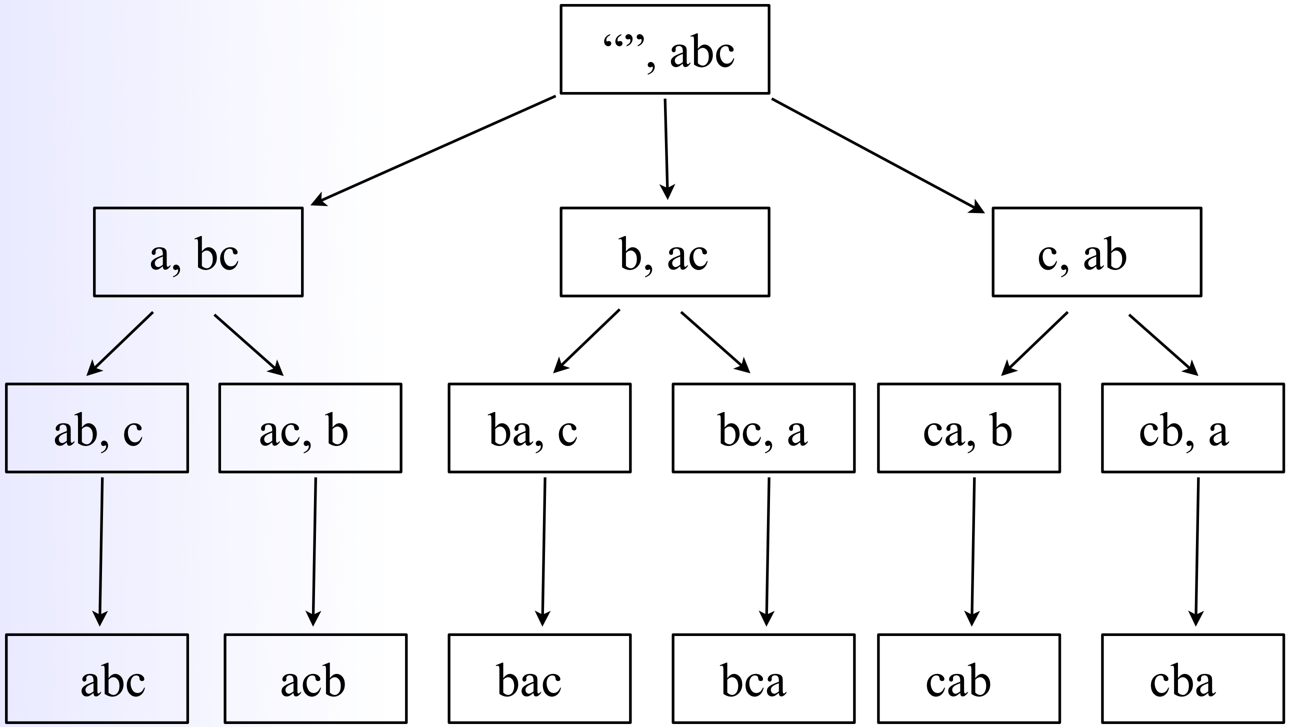
  - partial solution s=<c,ab> generates 2 new solutions <ca,b> and <cb,a>; they are put in S
  - S = {<cb, a>, <ca,b>, <b,ac>, <a,bc>}
  - …

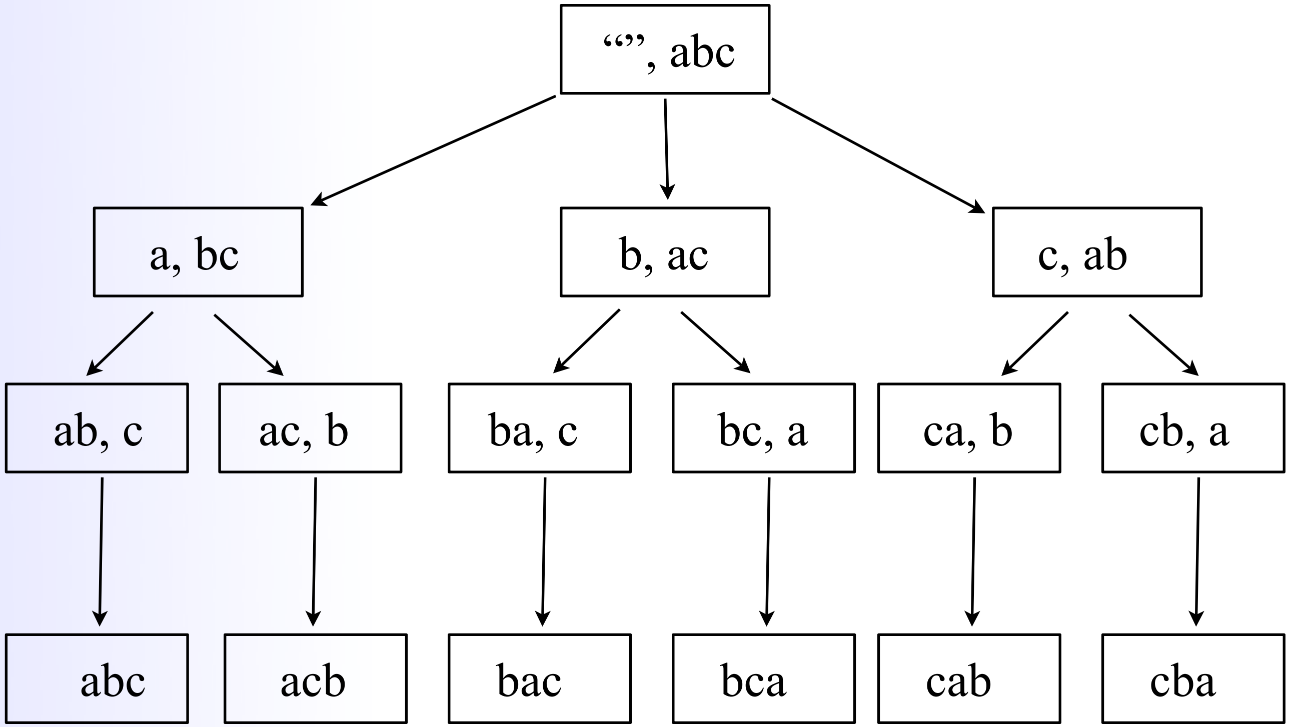
The solution space

- How does the algorithm traverse and construct the solution space when S is a stack?



The solution space

- Using a stack mimics recursion <----- goes depth first
  - depth-first search (DFS)
- Using a queue goes level by level <----- goes breadth first
  - breadth-first search (BFS)



Example: The missionary and cannibal problem

- You have 3 missionaries, 3 cannibals and a boat sitting on, say, the left side of a river.
- They all need to cross to the other side.
- Find a set of moves that brings all 6 people on the other side safely.
    - The boat can take at most two people at a time (and at least one).
    - Anybody can row
    - If at any point there are more cannibals than missionaries, the missionaries get eaten.

Missionaries and Cannibals

- We want to frame it as a search in a solution space and use the previous skeleton
- How to encode a state?
  - write a class MCState
- What's the initial state?
- What's the final state?
  - write MCState:isFinal()
- When is a state valid?
  - write MCState:isValid()
- Given a state, what are the moves you can make ?
- What will the set S contain?

Missionaries and Cannibals

- Queue<MCState> s = new Queue<MCState>();
- //add initial state
- s.insert(newMCState(3,3,0,0,1));
- while (!s.isEmpty())) {
  - MCState crt = s.delete();
  - if (crt.isFinal()) { //this is the goal state; break;}
  - //generate all possible next states and call s.insert() to add them to s
  - ...
- }
- //crt must be the final state; print it

- Are there duplicate states in S?
- Can a state be inserted in S several times? (This would correspond to a loop --- we go back to a state that we already explored). Why is this not a problem?
- The skeleton above uses a Queue for S. Would a Stack work? Why (not)?