csci 210: Data Structures

Recursion

Summary

- Topics
  - recursion overview
  - simple examples
  - Sierpinski gasket
  - Hanoi towers
  - Blob check

- READING:
  - GT textbook chapter 3.5

Recursion

- In general, a method of defining a function in terms of its own definition
  - f (n) = f(n-1) + f(n-2)
  - f(0) = f(1) = 1;
- In programming, recursion is a call to the same method from a method
- Why write a method that calls itself?
  - a method to to solve problems by solving easier instance of the same problem
- Recursive function calls can result in a an infinite loop of calls
  - recursion needs a base-case in order to stop
  - f(0) = f(1) = 1;

- Recursion (repetitive structure) can be found in nature
  - shape of cells, leaves

- Recursion is a good problem solving approach
- Recursive algorithms
  - elegant
  - simple to understand and prove correct
  - easy to implement

- Problem solving technique: Divide-and-Conquer
    - break into smaller problems
    - solve sub-problems recursively
    - assemble solutions

- recursive-algorithm(input) {
    - //base-case
    - if (isSmallEnough(input))
        - compute the solution and return it
    - else
        - break input into simpler instances input1, input 2,...
        - solution1 = recursive-algorithm(input1)
        - solution2 = recursive-algorithm(input2)
        - ...
        - figure out solution to this problem from solution1, solution2,...
        - return solution
- }
-

- Problem: write a function that computes the sum of numbers from 1 to n

```
int sum (int n)
```

1. use a loop
2. recursively

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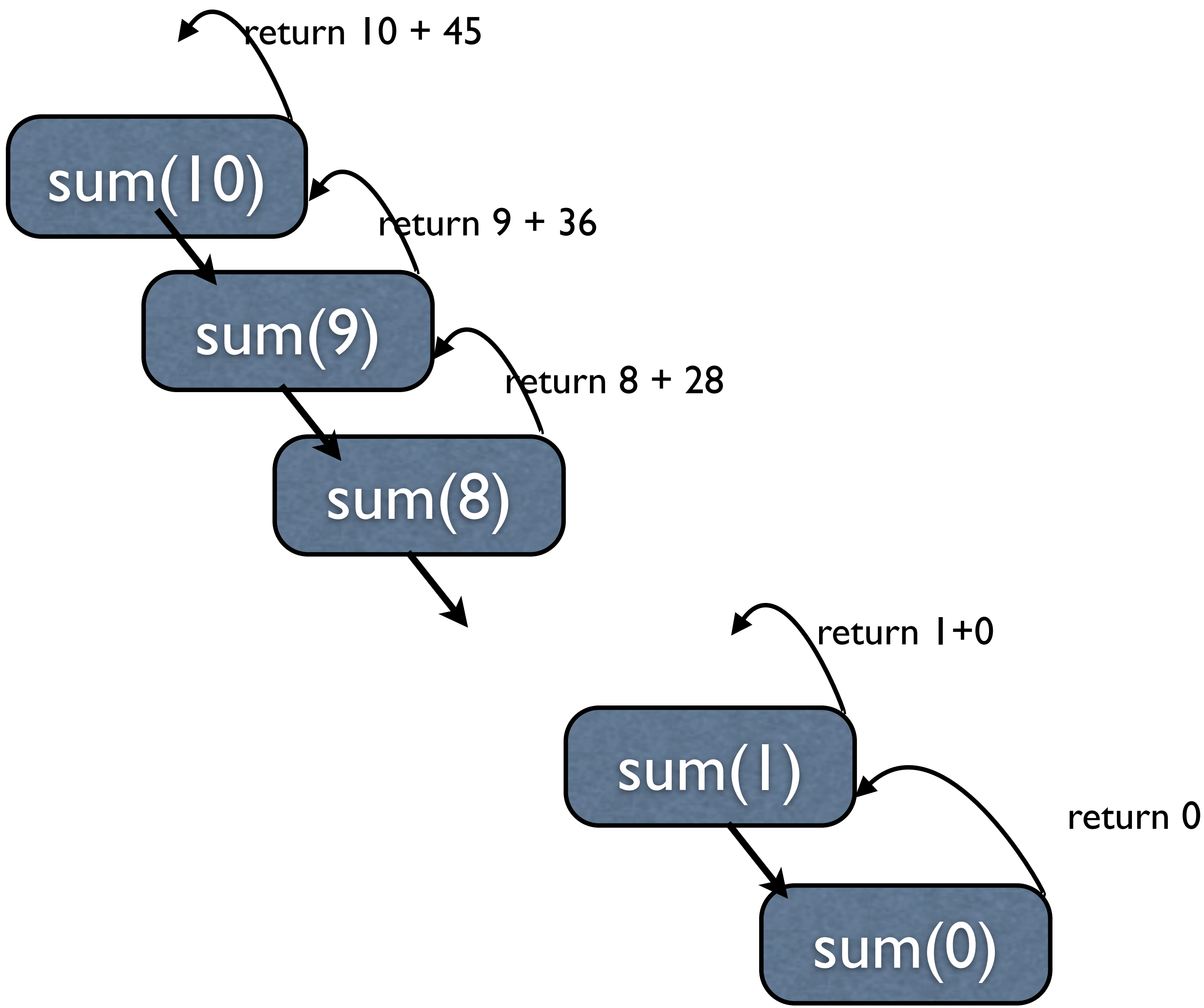
1. use a loop
2. recursively

```
int sum (int n) {
    int s = 0;
    for (int i=0; i<n; i++)
        s+= i;
    return s;
}
```

```
int sum (int n) {
    int s;
    if (n == 0) return 0;
     //else
     s = n + sum(n-1);
     return s;
}
```

How does it work?


return 10 + 45
sum(10)
return 9 + 36
sum(9)
return 8 + 28
sum(8)
return 1+0
sum(1)
return 0
sum(0)

Recursion

- How it works
  - Recursion is no different than a function call
  - The system keeps track of the sequence of method calls that have been started but not finished yet (active calls)
    - order matters

- Recursion pitfalls
  - miss base-case
    - infinite recursion, stack overflow
  - no convergence
    - solve recursively a problem that is not simpler than the original one

Perspective

- Recursion leads to
  - compact
  - simple
  - easy-to-understand
  - easy-to-prove-correct
- solutions

- Recursion emphasizes thinking about a problem at a high level of abstraction

- Recursion has an overhead (keep track of all active frames). Modern compilers can often optimize the code and eliminate recursion.

- First rule of code optimization:
  - Don’t optimize it..yet.

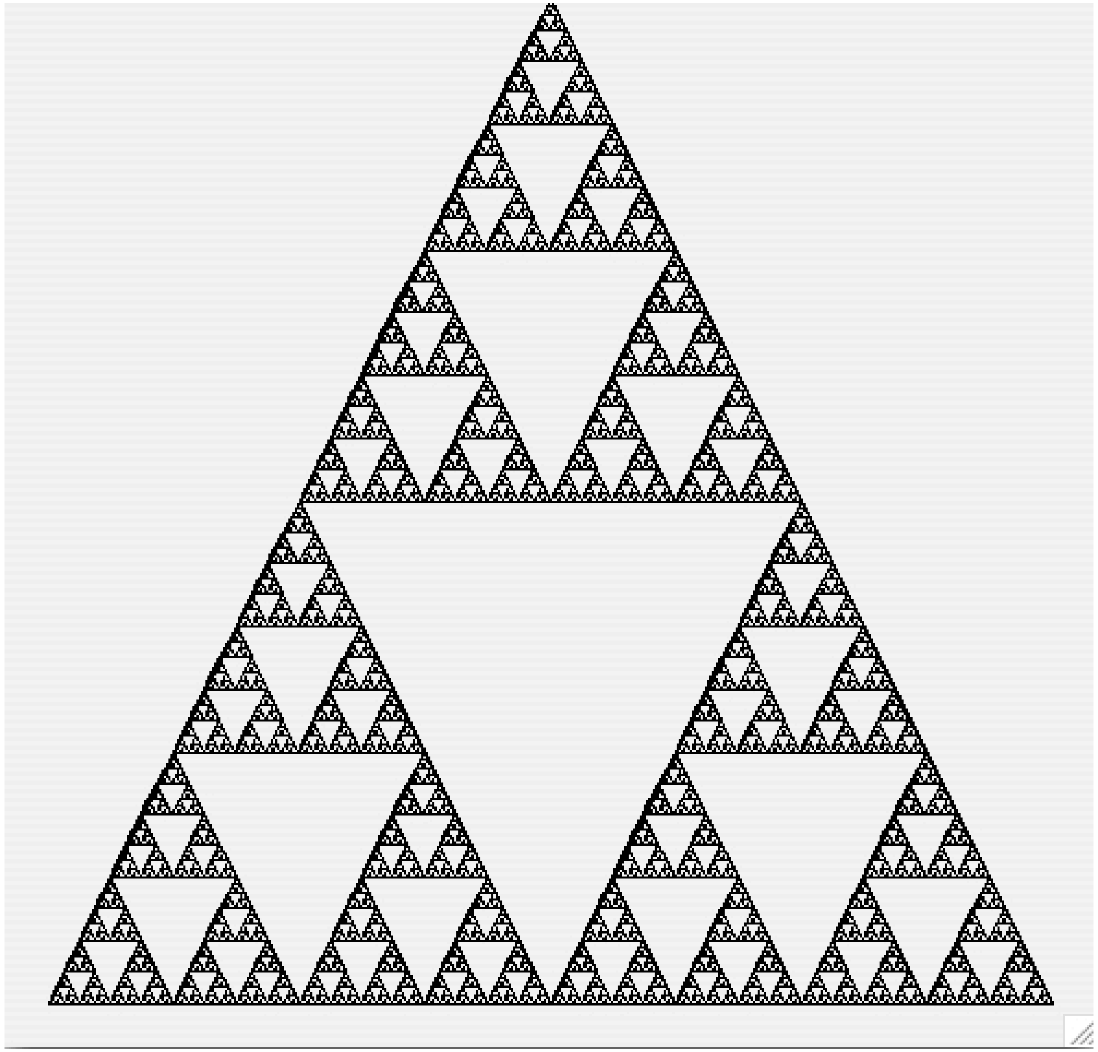
- Unless you write super-duper optimized code, recursion is good

- Mastering recursion is essential to understanding computation.

Class-work

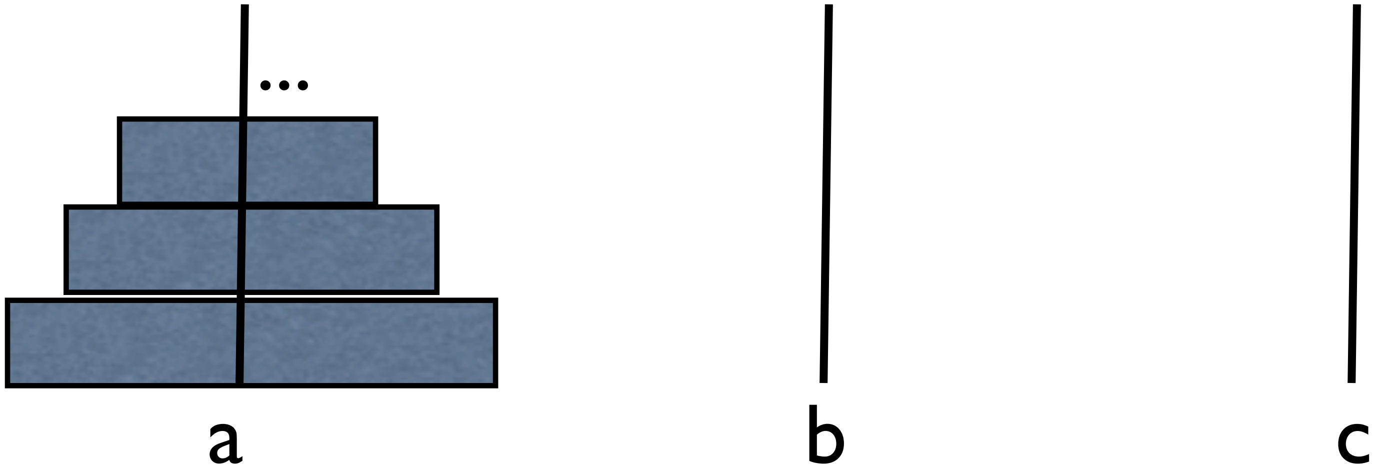
- Sierpinski gasket

- Fill in the code to create this pattern

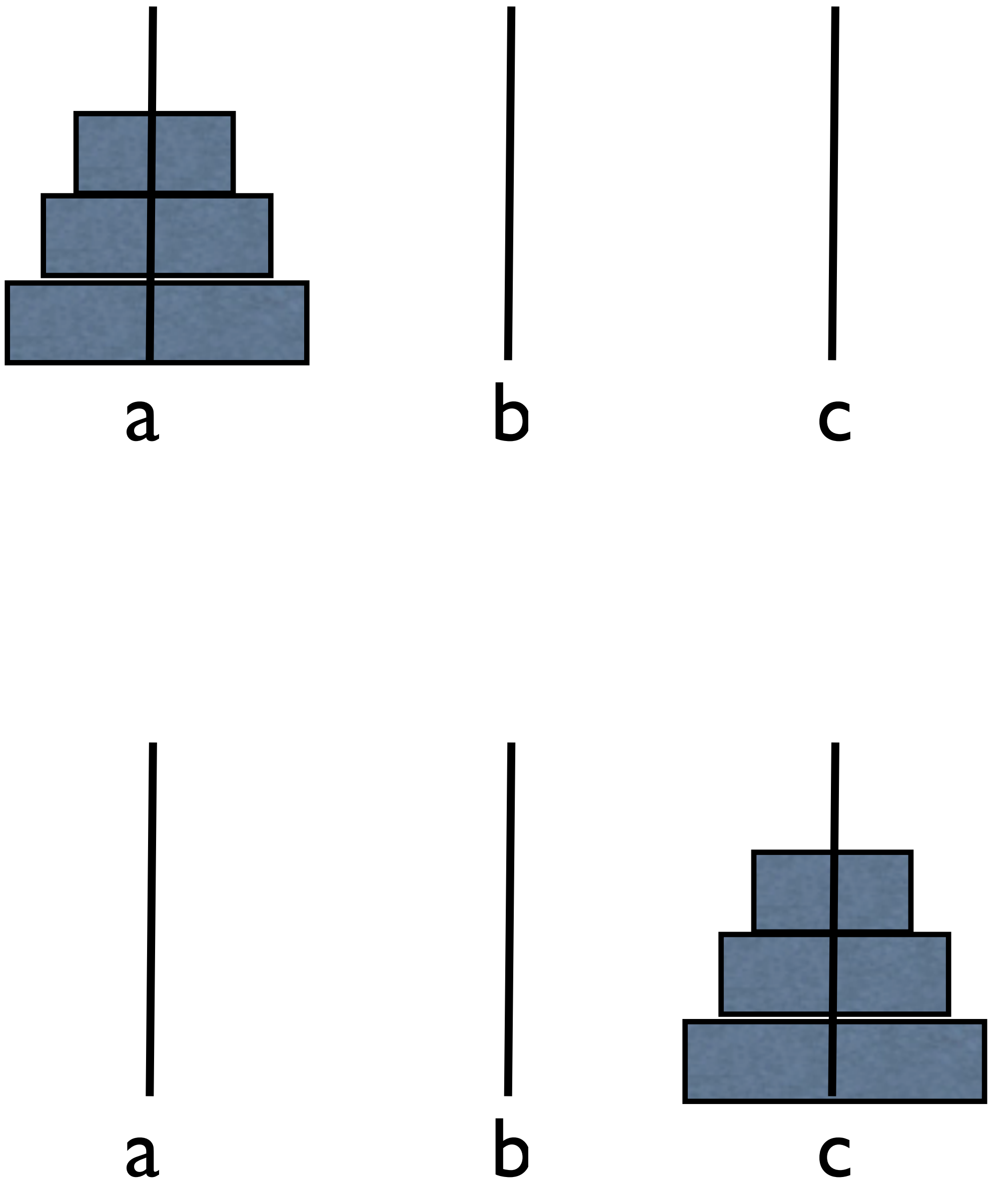


Towers of Hanoi

- Consider the following puzzle
  - There are 3 pegs (posts) a, b, c
  - n disks of different sizes
  - each disk has a hole in the middle so that it can fit on any peg
  - at the beginning of the game, all n disks are on peg a, arranged such that the largest is on the bottom, and on top sit the progressively smaller disks, forming a tower
  - Goal: find a set of moves to bring all disks on peg c in the same order, that is, largest on bottom, smallest on top
    - constraints
      - the only allowed type of move is to grab one disk from the top of one peg and drop it on another peg
      - a larger disk can never lie above a smaller disk, at any time
- PS: the legend says that the world will end when a group of monks, somewhere in a temple, will finish this task with 64 golden disks on 3 diamond pegs. Not known when they started.



Find the set of moves for n=3



Solving the problem for any n

- Think recursively
- Problem: move n disks from A to C using B
- Can you express the problem in terms of a smaller problem?
- Subproblem:
  - move n-1 disks from A to C using B

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- Recursive formulation of Towers of Hanoi

- move n disks from A to C using B
  - move top n-1 disks from A to B
  - move bottom disks from A to C
  - move n-1 disks from B to C using A

- Correctness
  - How would you go about proving that this is correct?

Hanoi-skeleton.java

- Look over the skeleton of the Java program to solve the Towers of Hanoi
- It's supposed to ask you for n and then display the set of moves
  - no graphics

- finn in the gaps in the method

```
public void move(sourcePeg, storagePeg, destinationPeg)
```

Correctness

- Proving recursive solutions correct is related to mathematical induction, a technique of proving that some statement is true for any n
  - induction is known from ancient times (the Greeks)
- Induction proof:
  - Base case: prove that the statement is true for some small value of n, usually n=1
  - The induction step: assume that the statement is true for all integers <= n-1. Then prove that this implies that it is true for n.
- Exercise: try proving by induction that $1 + 2 + 3 + \ldots + n = n(n+1)/2$
- A recursive solution is similar to an inductive proof; just that instead of "inducting" from values smaller than n to n, we "reduce" from n to values smaller than n (think n = input size)
  - the base case is crucial: mathematically, induction does not hold without it; when programming, the lack of a base-case causes an infinite recursion loop
- proof sketch for Towers of Hanoi:
  - It works correctly for moving one disk (base case). Assume it works correctly for moving n-1 disks. Then we need to argue that it works correctly for moving n disks.

Analysis

- How close is the end of the world?

- Let's estimate running time

- the running time of recursive algorithms is estimated using recurrent functions
- let T(n) be the time it takes to compute the sequence of moves to move n disks fromont peg to another
- then based on the algorithm we have
  - $T(n) = 2T(n-1) + 1$, for any $n > 1$
  - $T(1) = 1$ (the base case)

- I can be shown by induction that $T(n) = 2^n - 1$
  - the running time is exponential in n

- Exercise:
  - 1GHz processor, n = 64 => $2^{64} \times 10^{-9}$ = ... a log time; hundreds of years