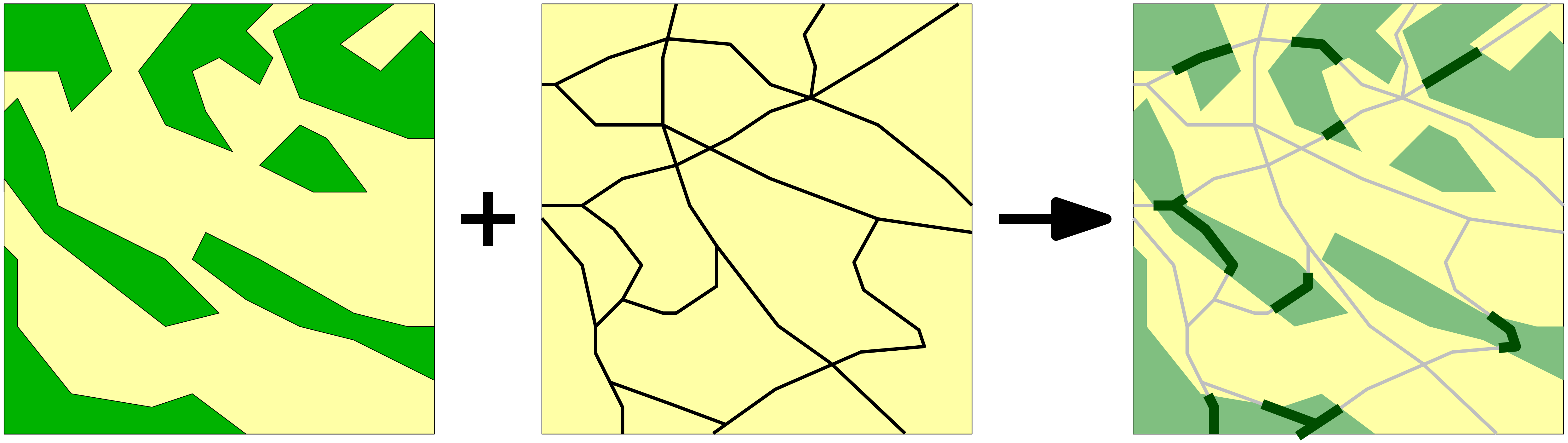
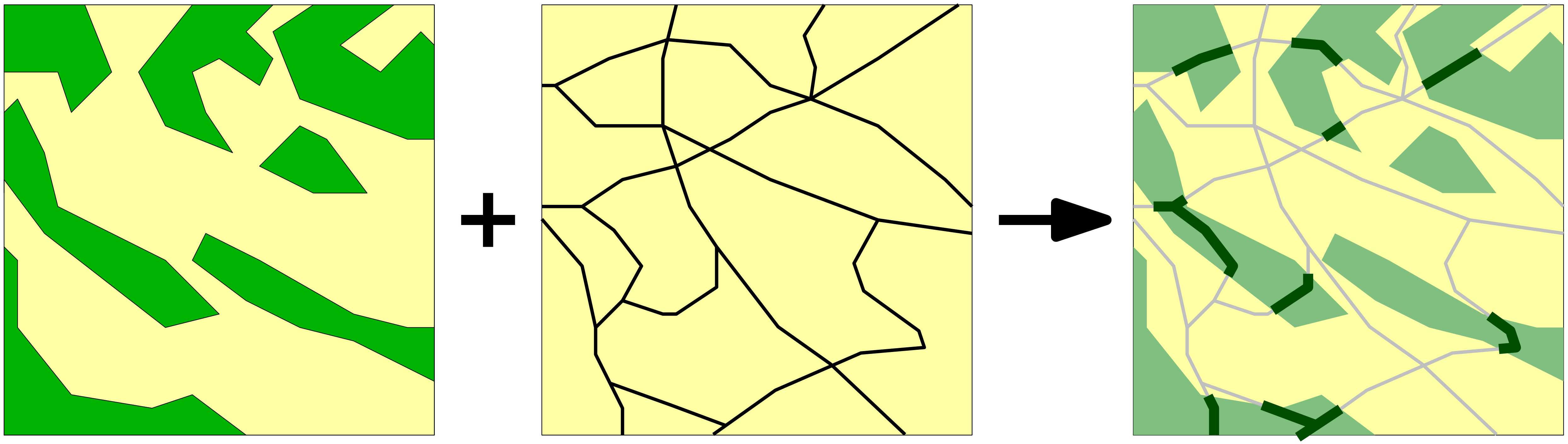
I/O-Efficient Map Overlay and Point Location in Low-Density Subdivisions

Mark de Berg Herman Haverkort Shripad Thite Laura Toma



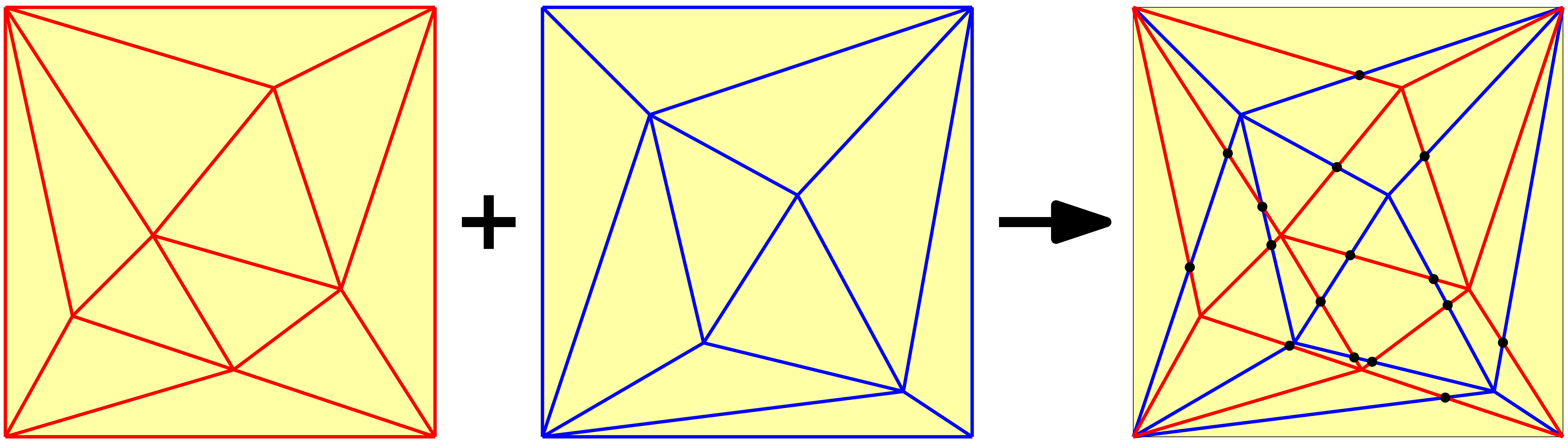
Maps: planar subdivisions, sets of (non-intersecting) line segments, ...



I/O-Efficient Map Overlay and Point Location on Low-Density Planar Maps

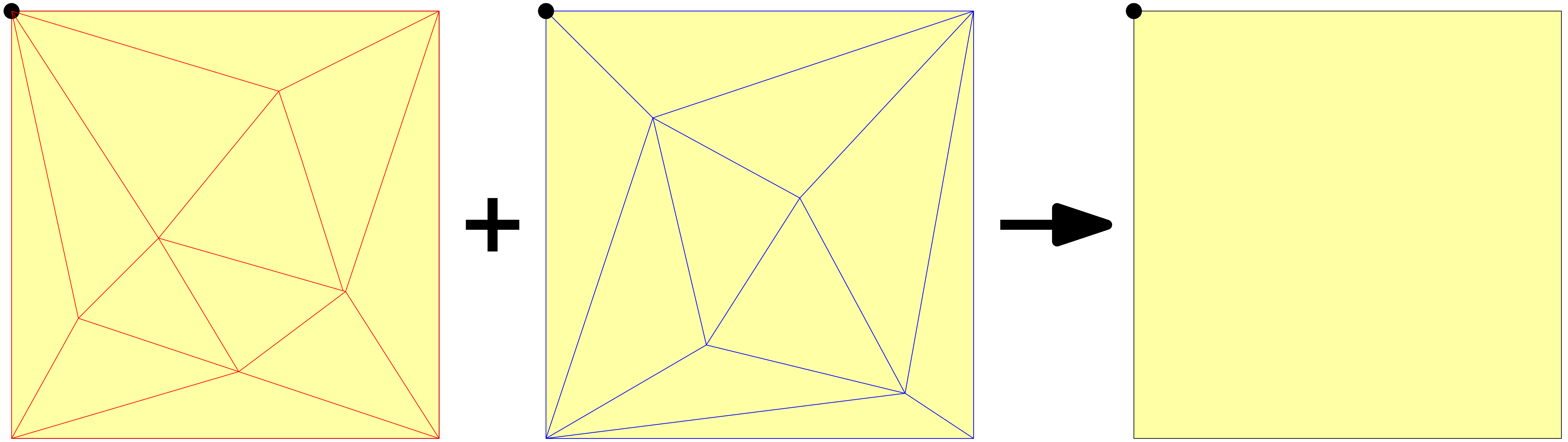
Mark de Berg Herman Haverkort Shripad Thite Laura Toma

Maps: ..., triangulations



Overlaying triangulations CPU-efficiently

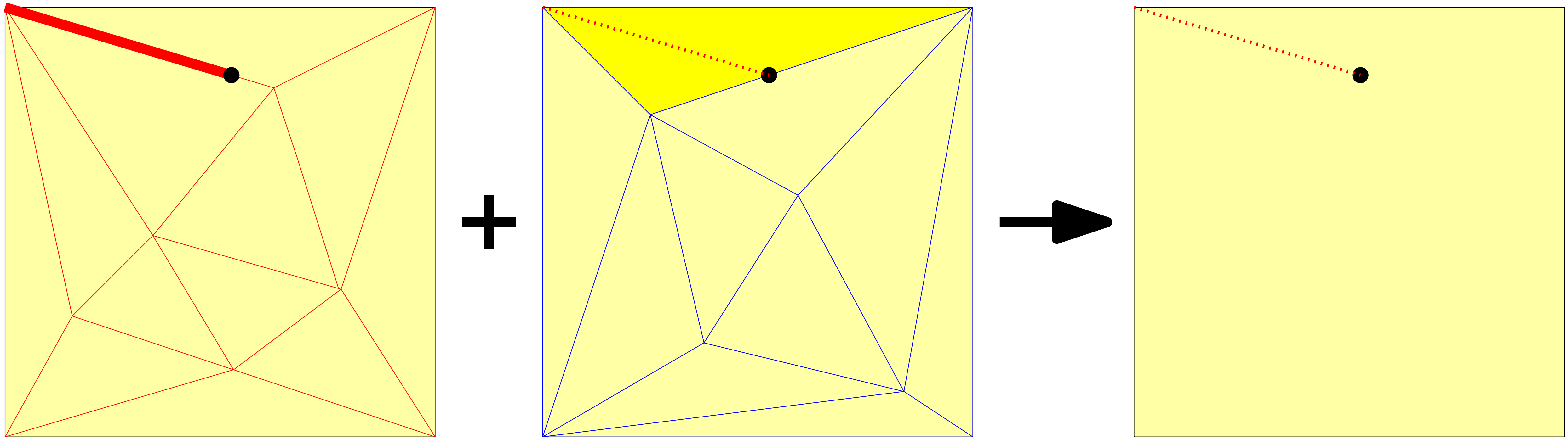
Maps: ..., triangulations



DFS in one triangulation, traverse triangles in the other

Overlaying triangulations CPU-efficiently

Maps: ..., triangulations



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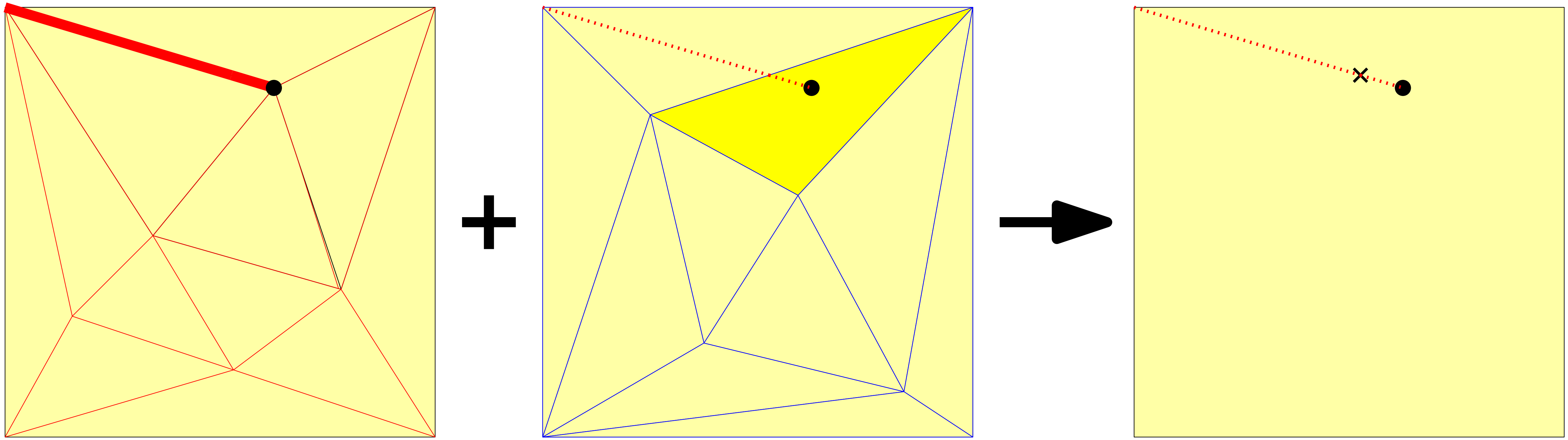

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Overlaying triangulations CPU-efficiently

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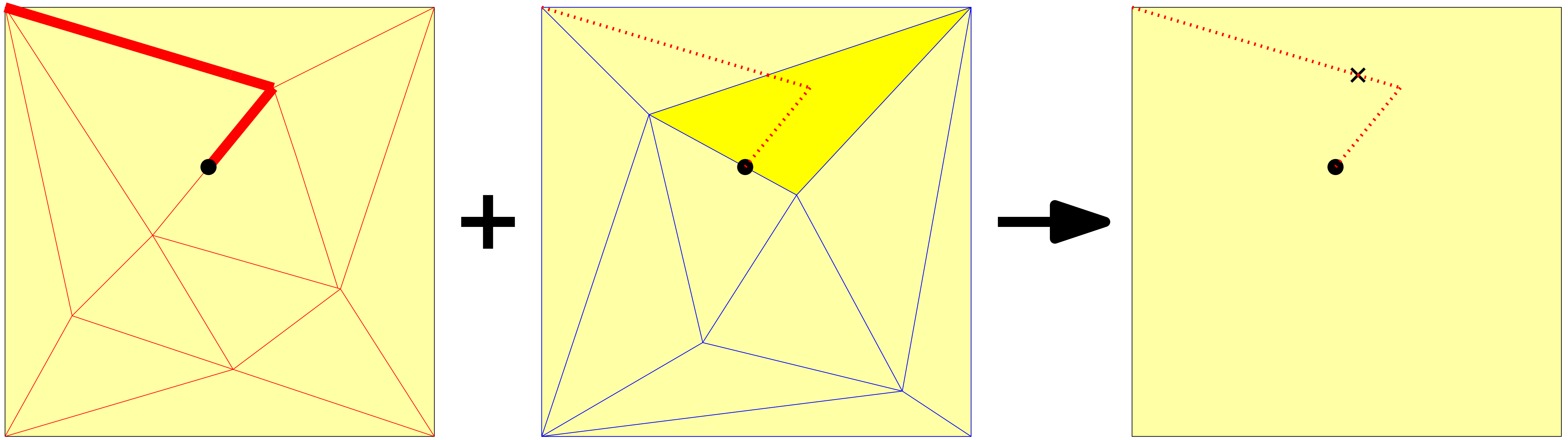

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Overlaying triangulations CPU-efficiently

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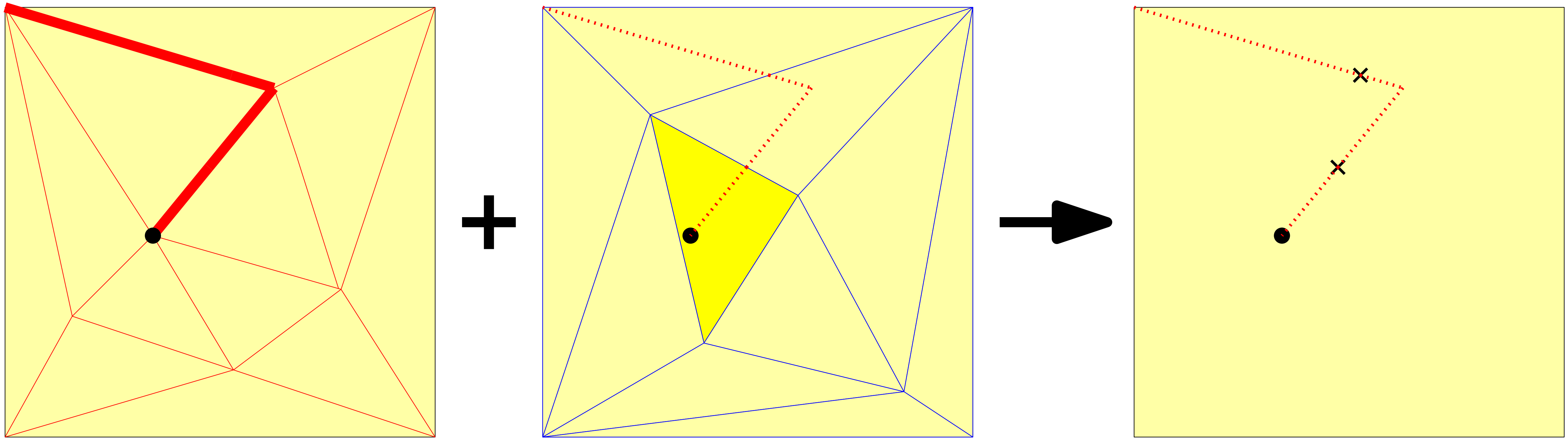

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Overlaying triangulations CPU-efficiently

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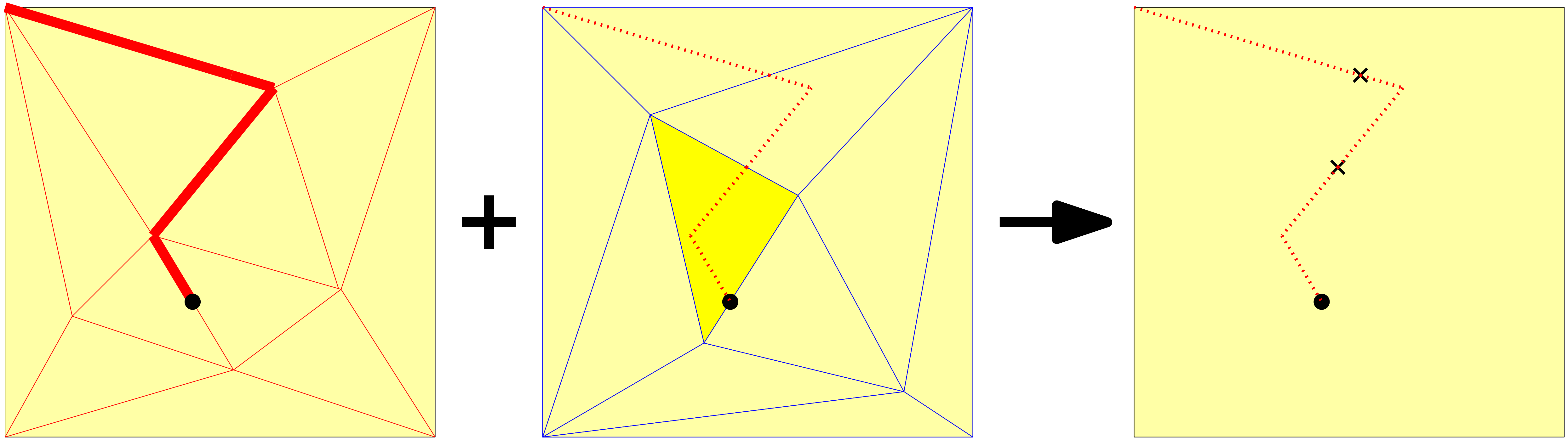

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Overlaying triangulations CPU-efficiently

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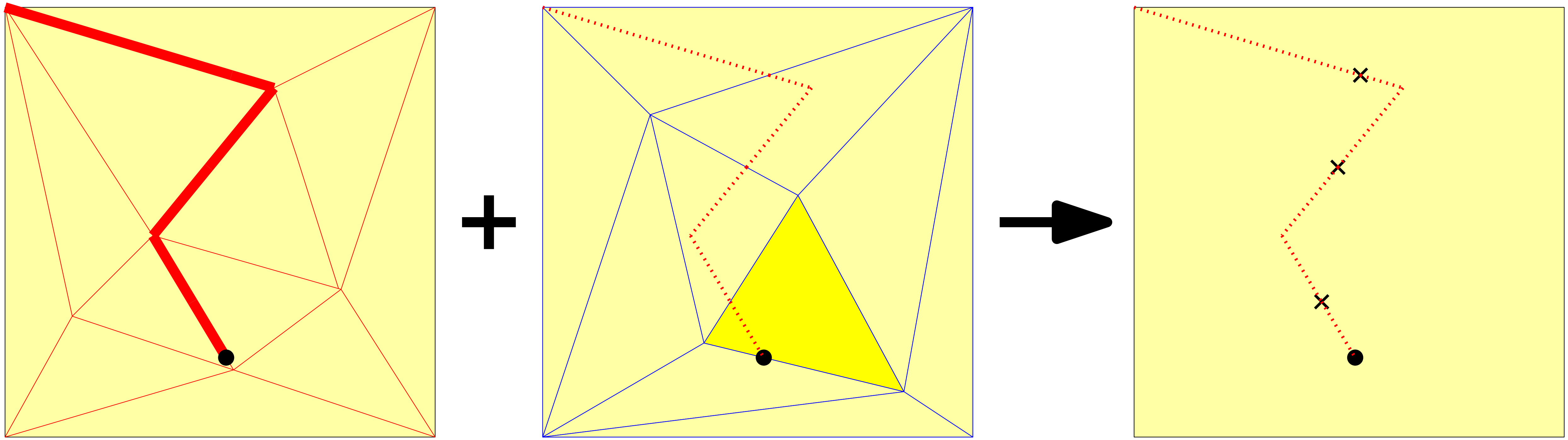

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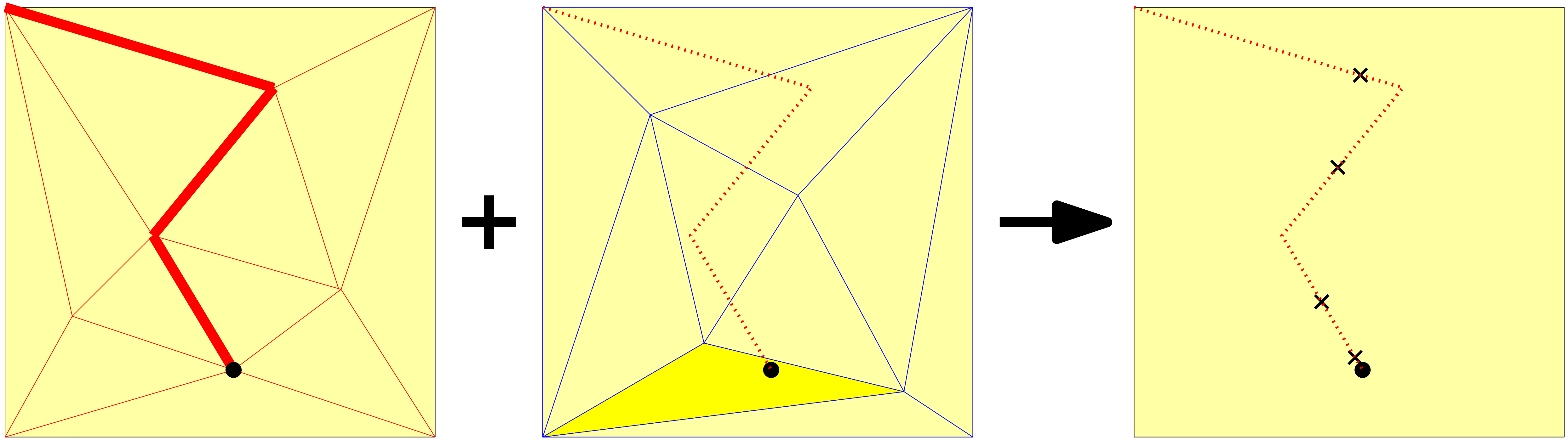

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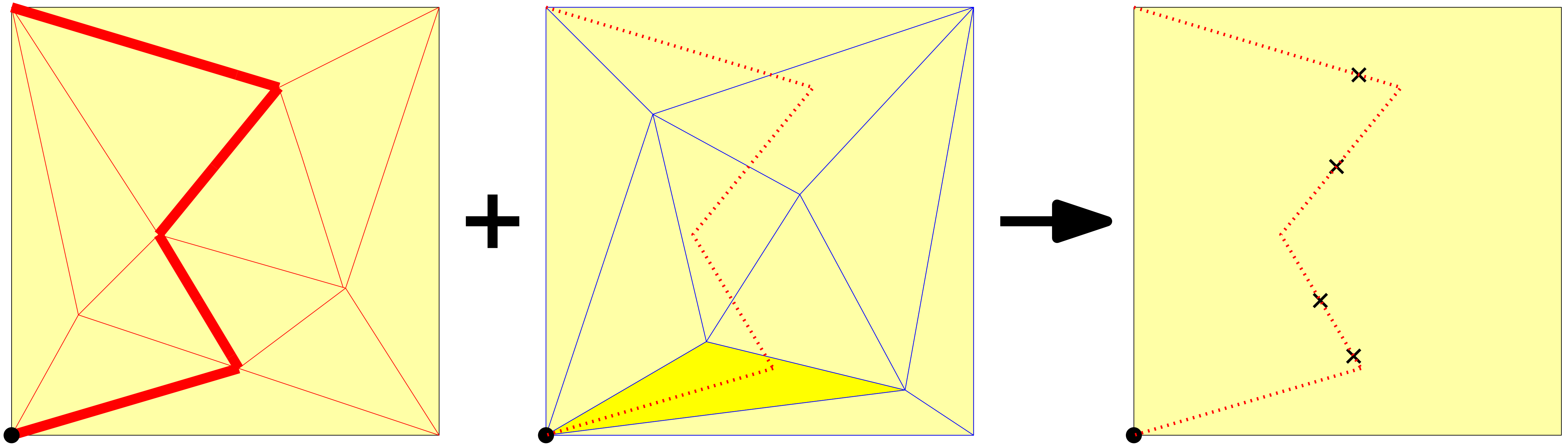

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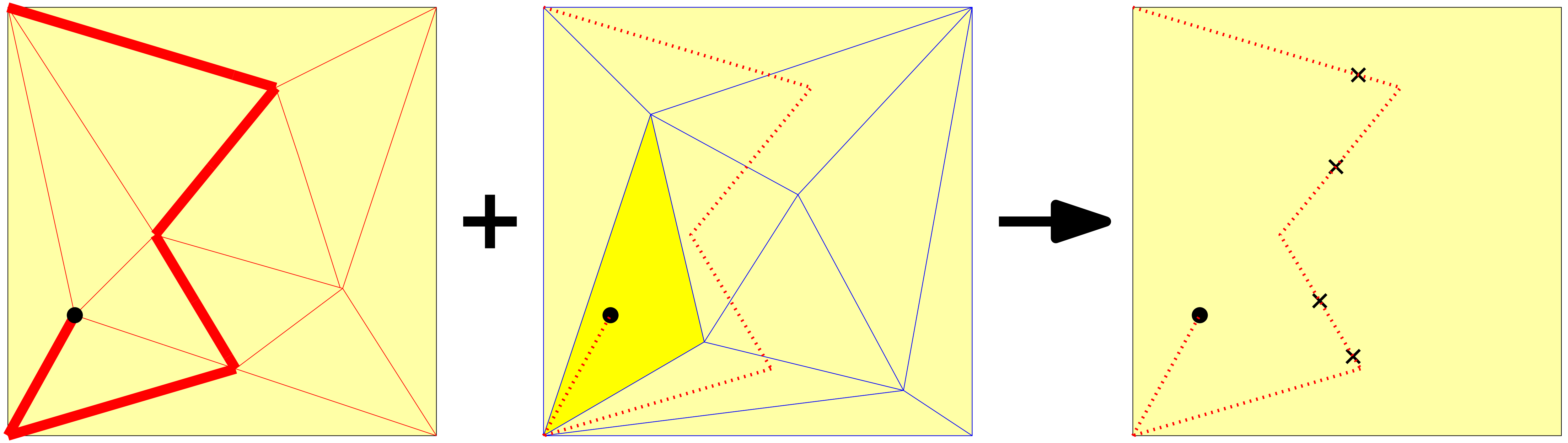

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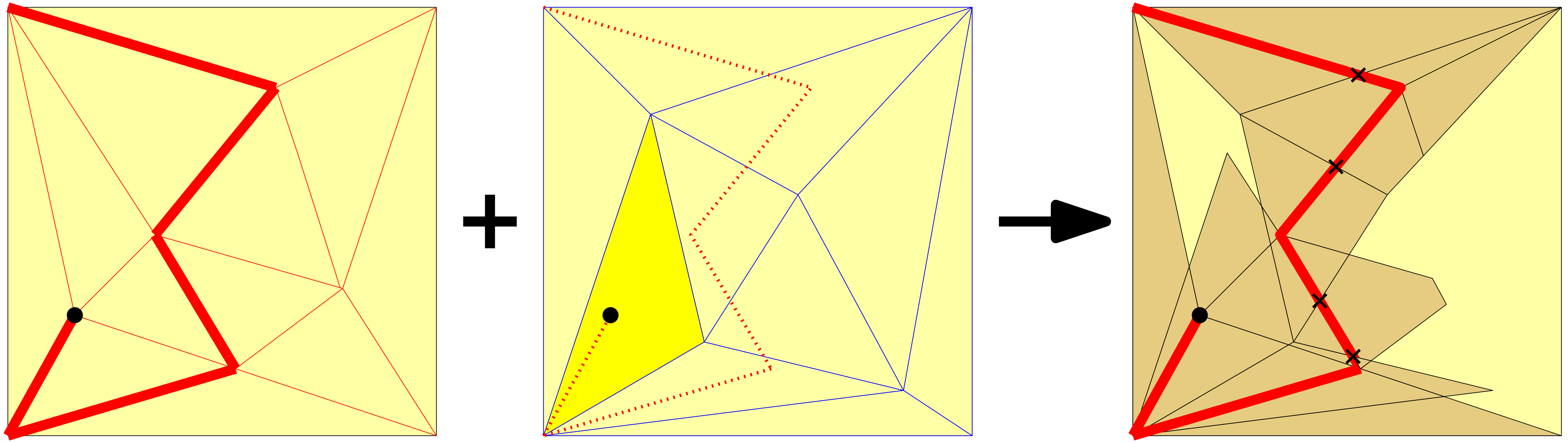

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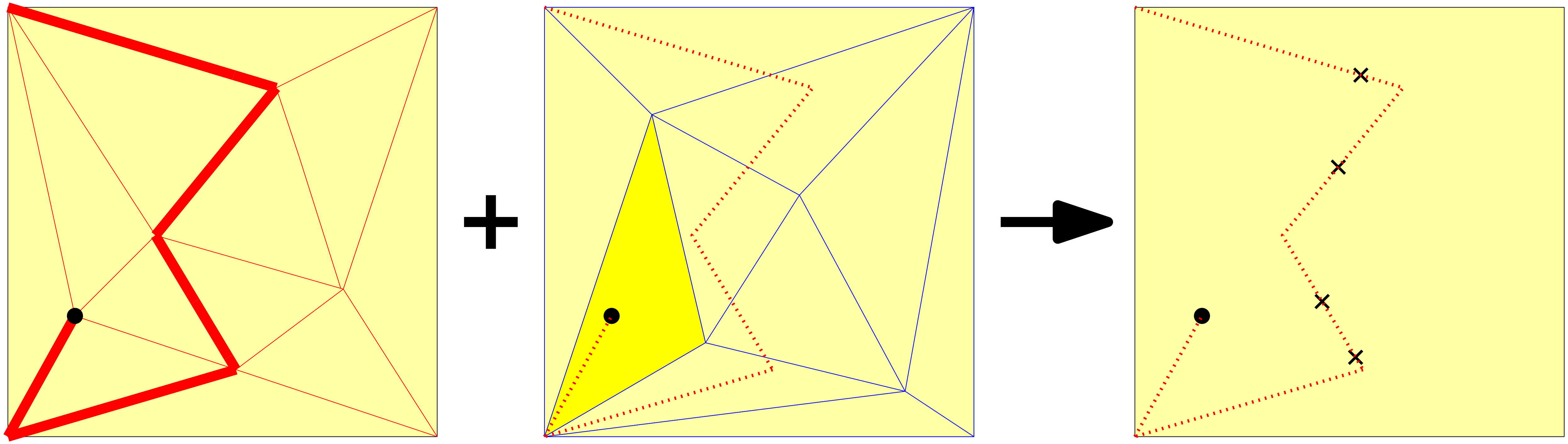


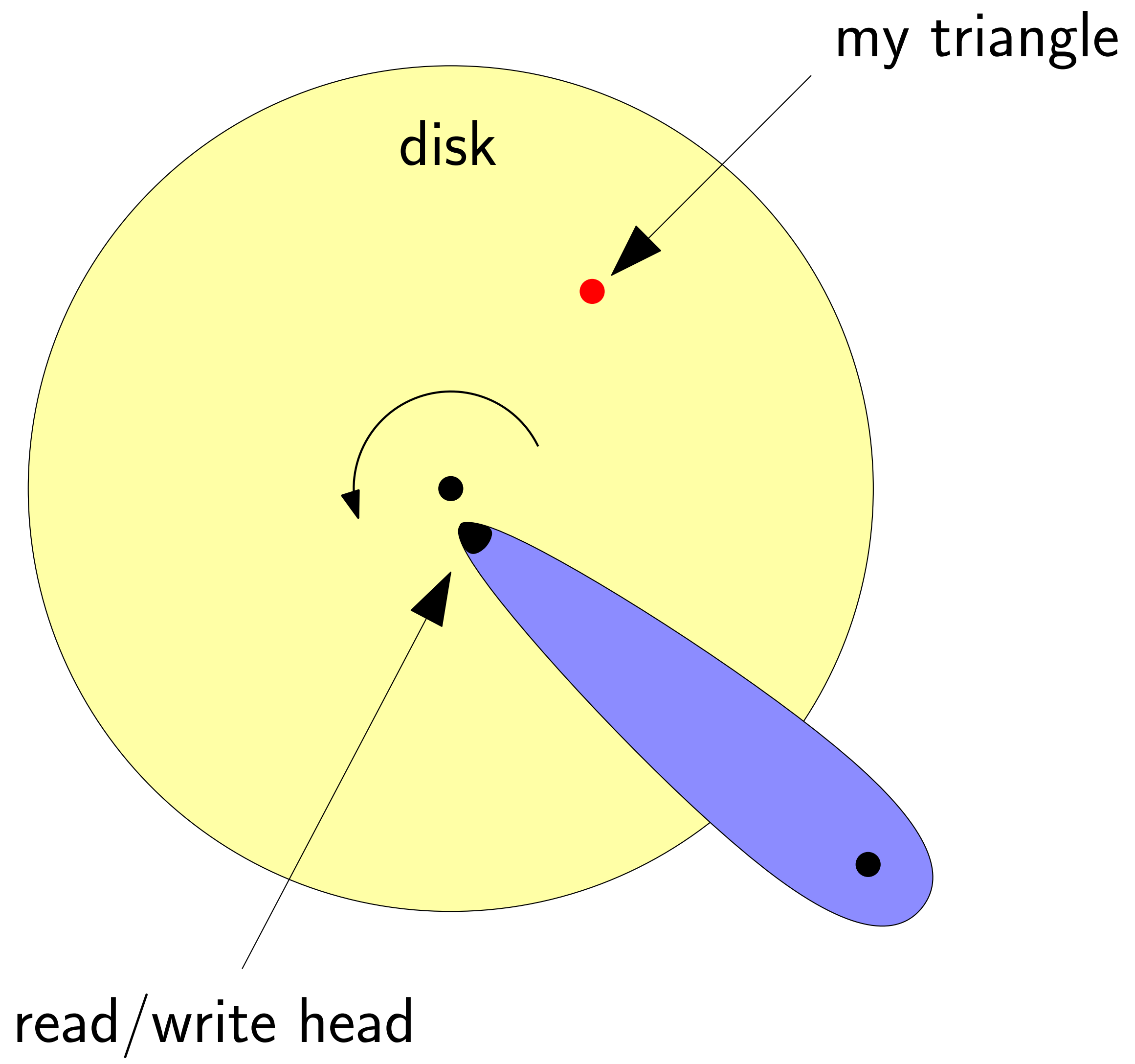
DFS in one triangulation, traverse triangles in the other:

- $\Theta(1)$ operations per edge
- $\Theta(1)$ operations per crossing

Total: $\Theta(n + k)$ CPU-operations (for n triangles, k intersections)

Using external memory

main memory
of size M
(too small for
all data)



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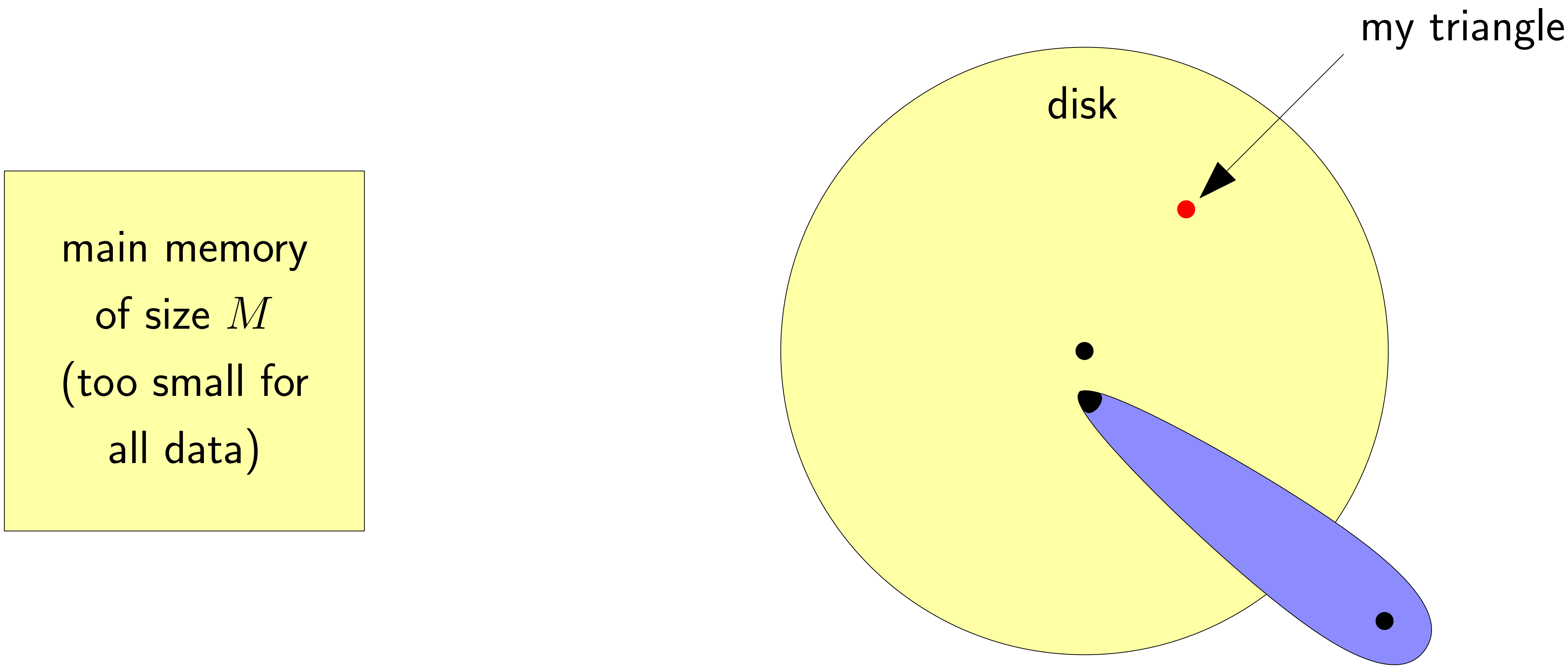

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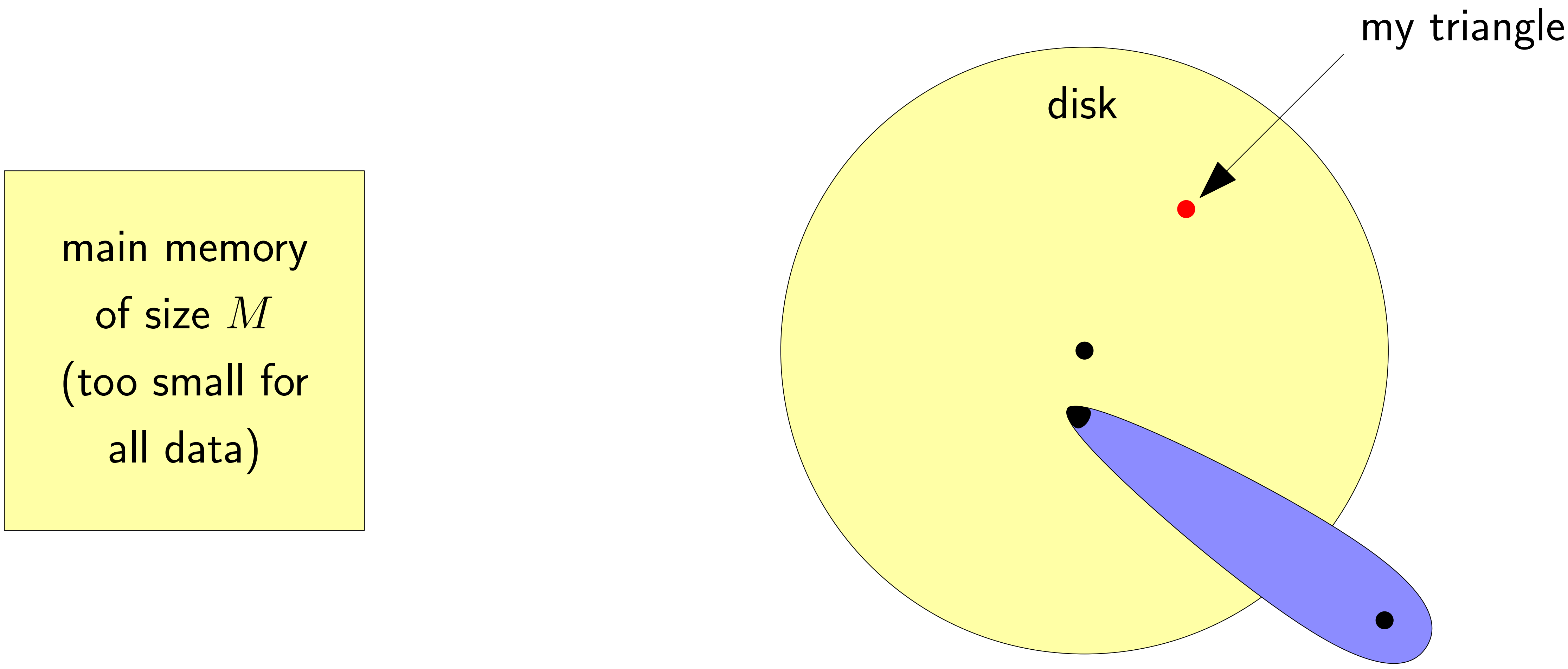

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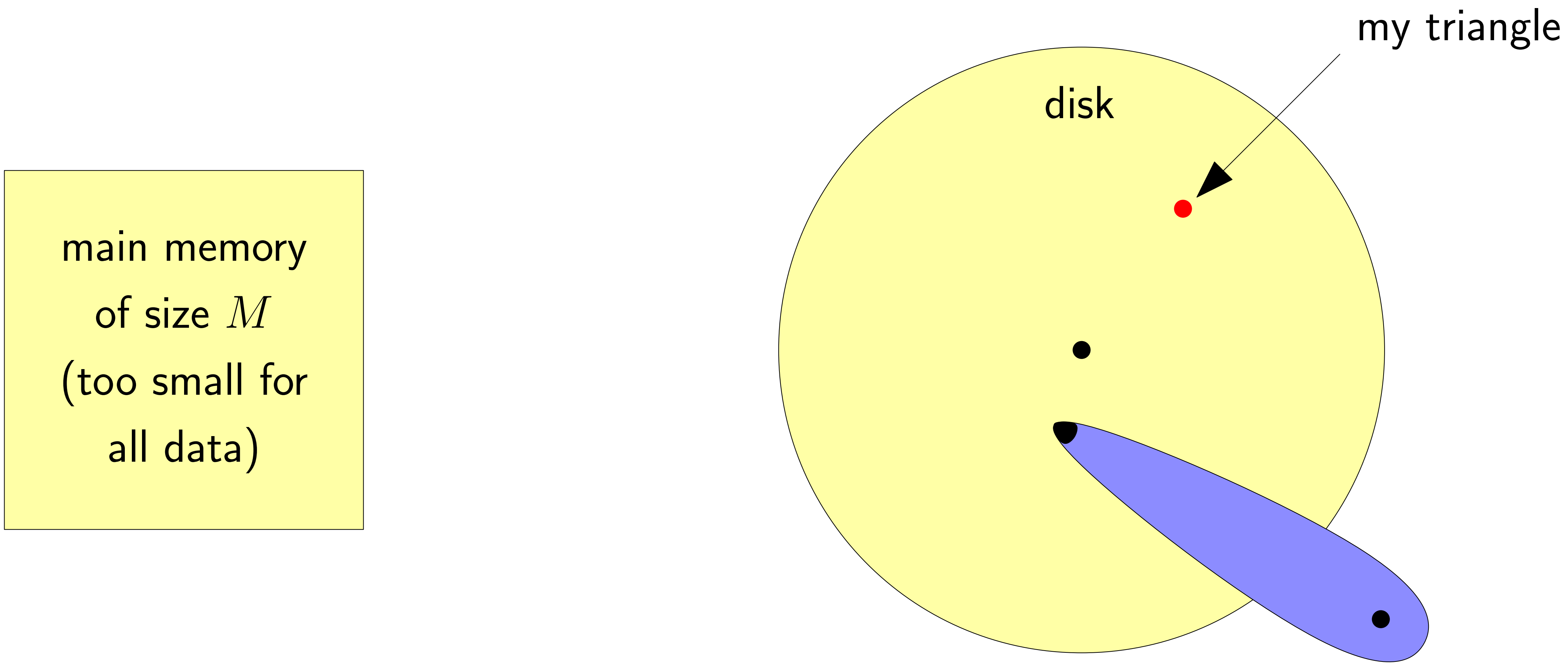

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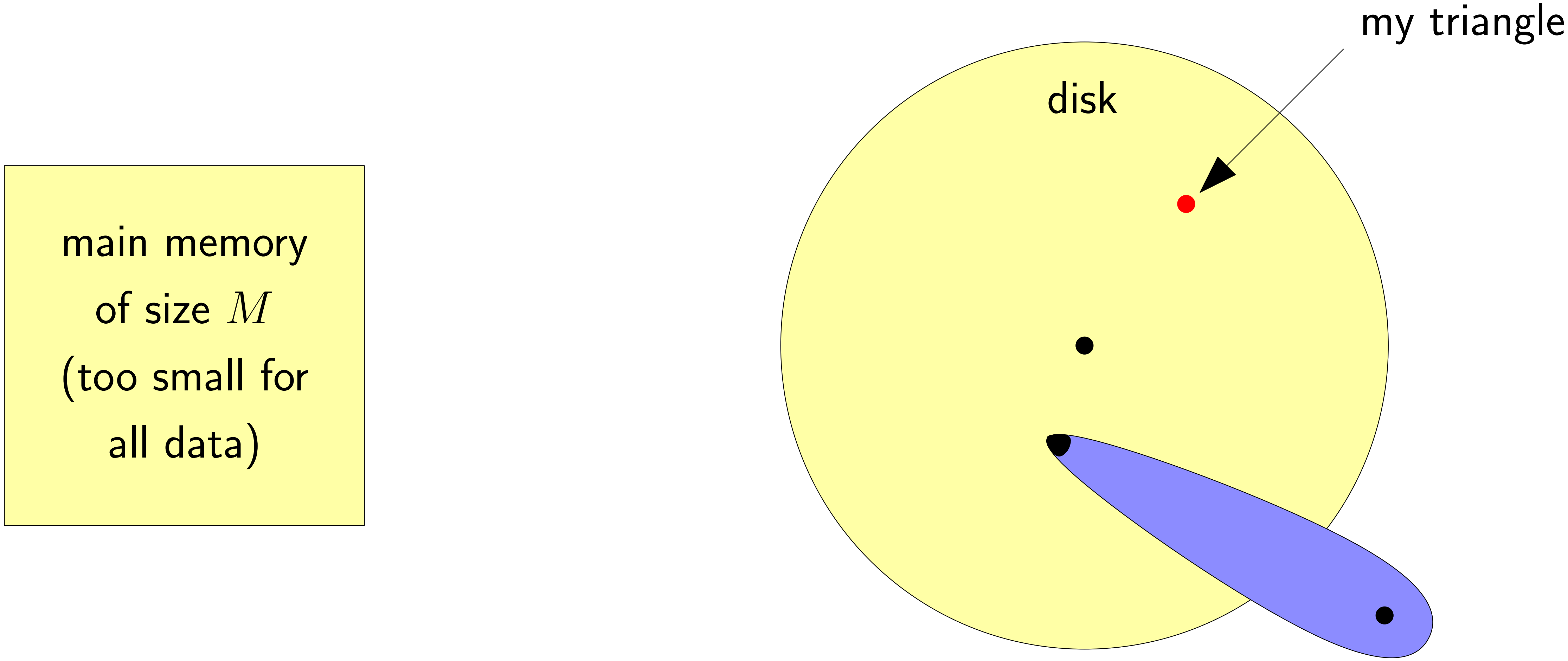

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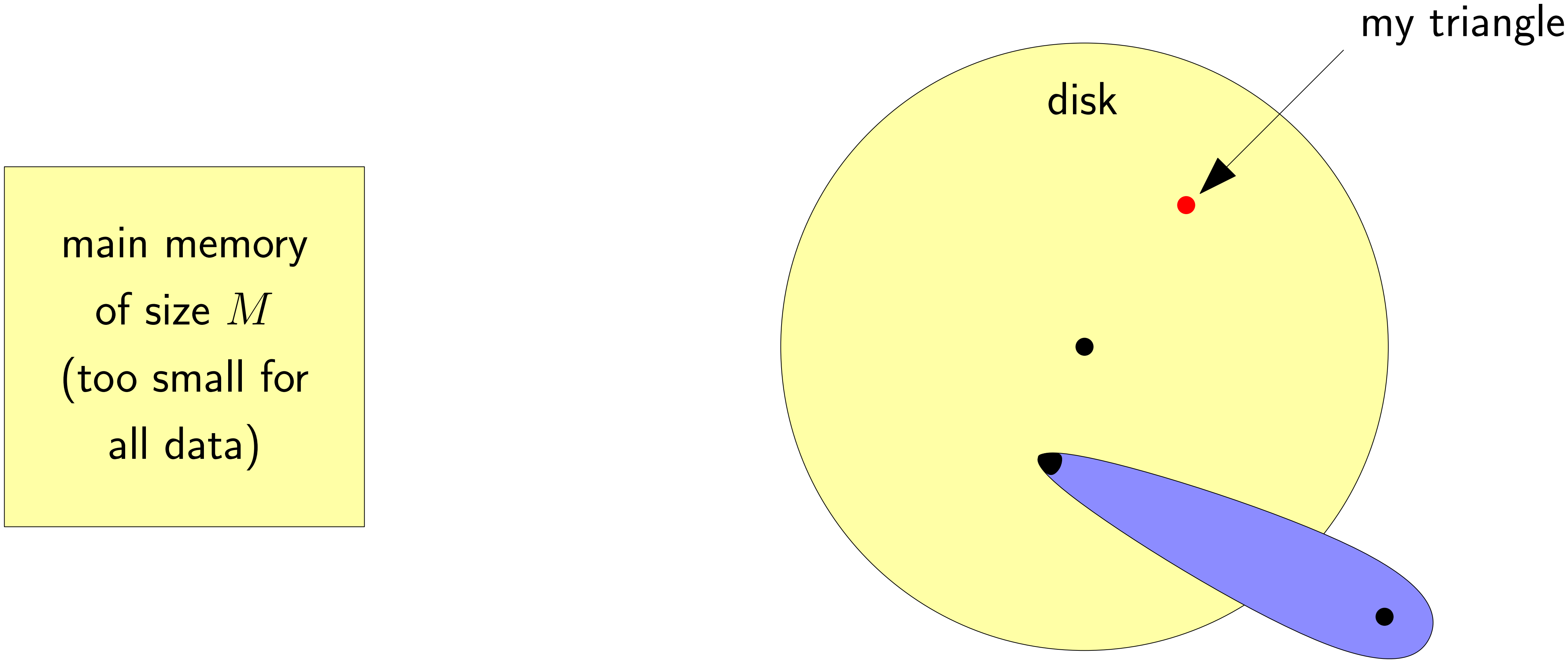

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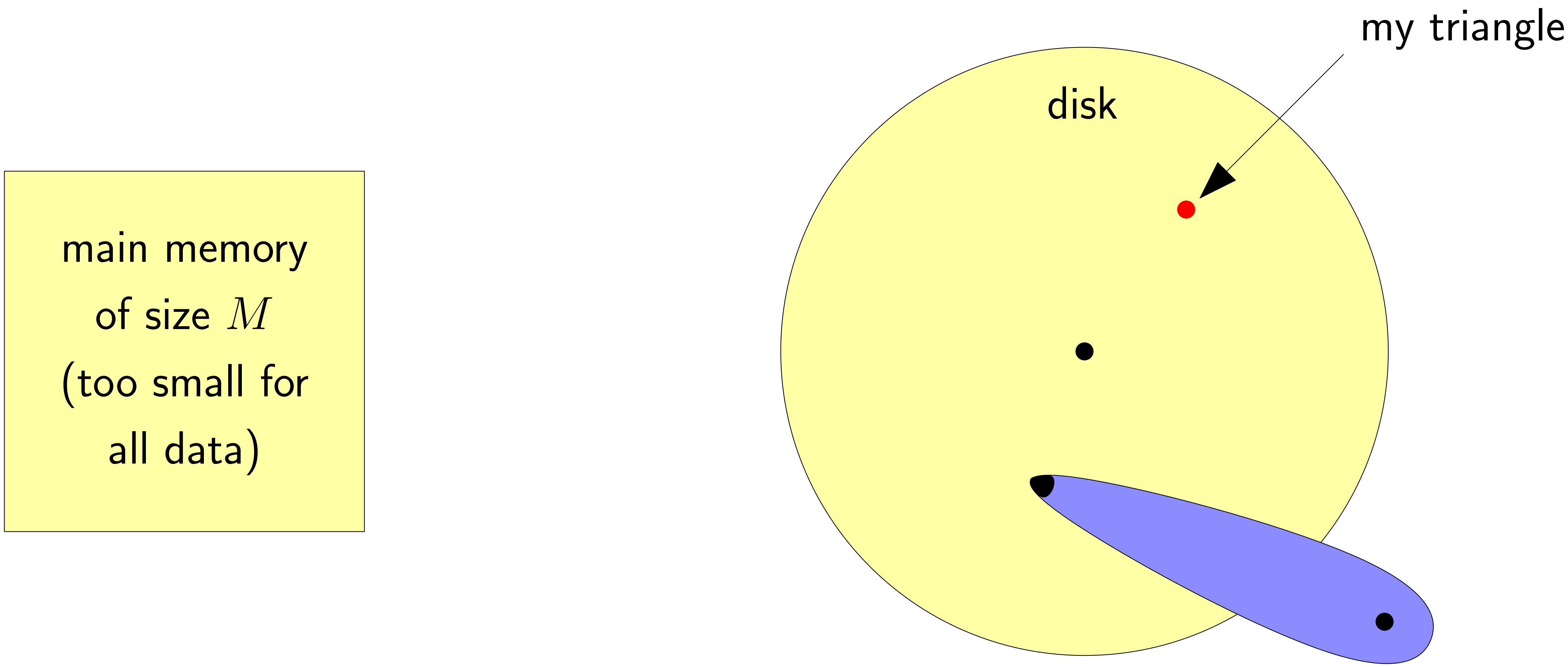

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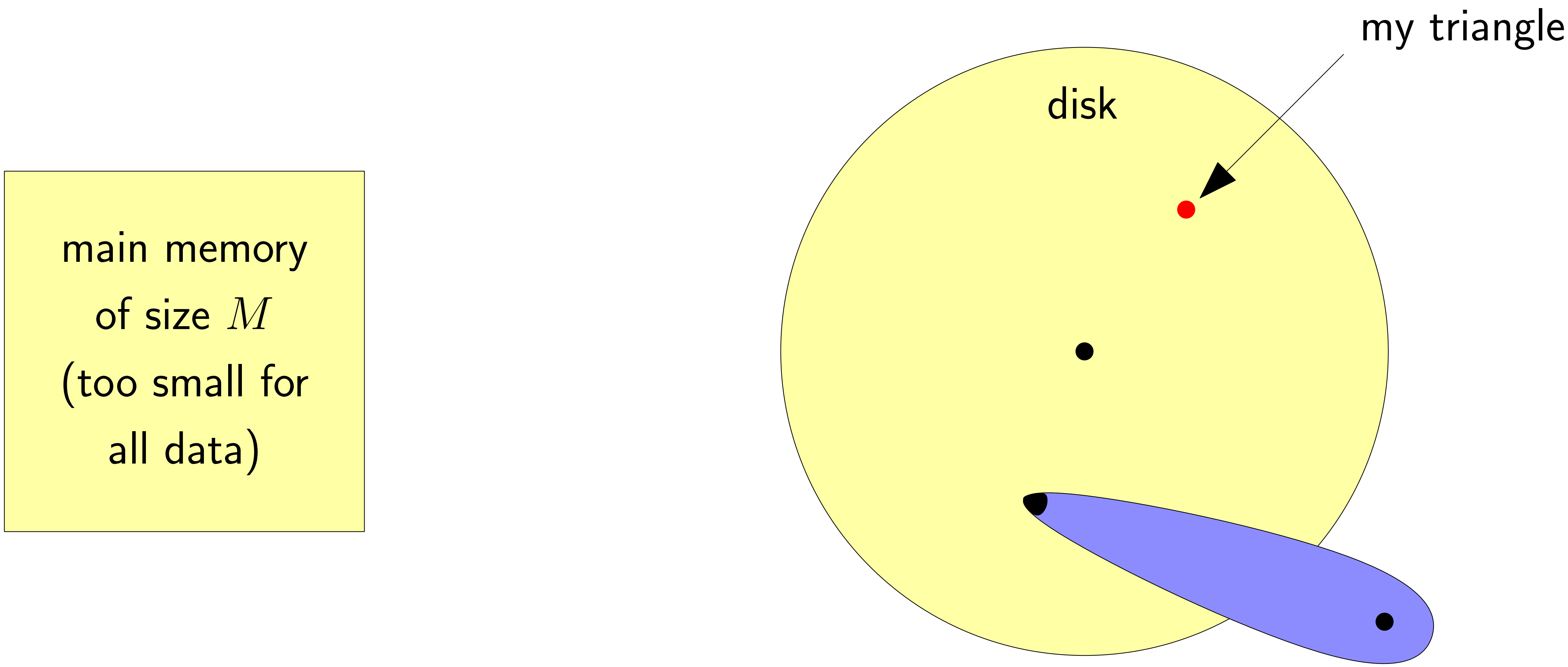

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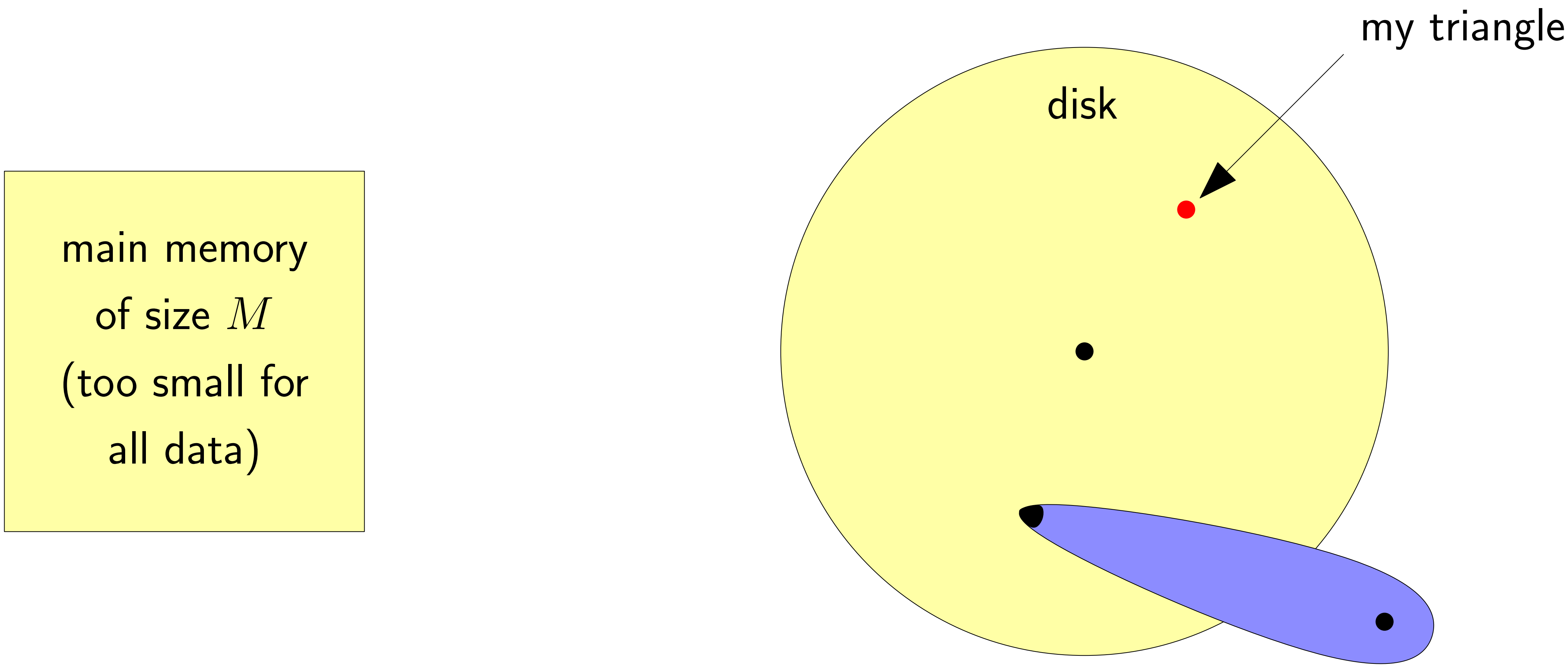

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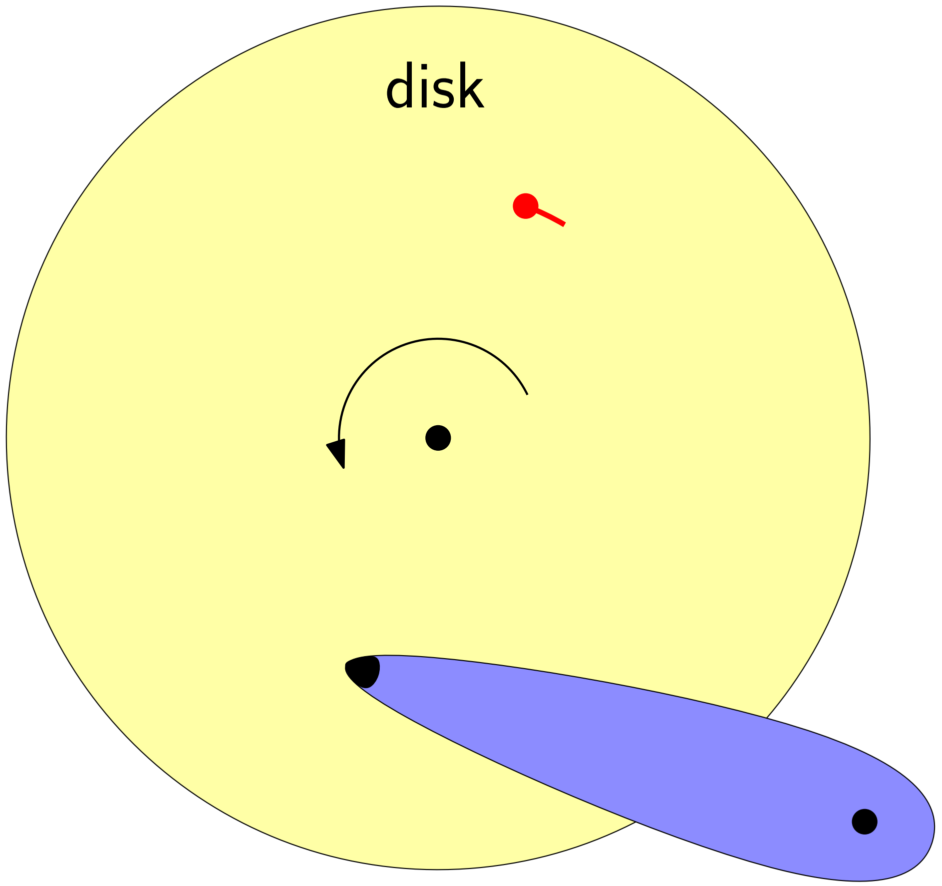

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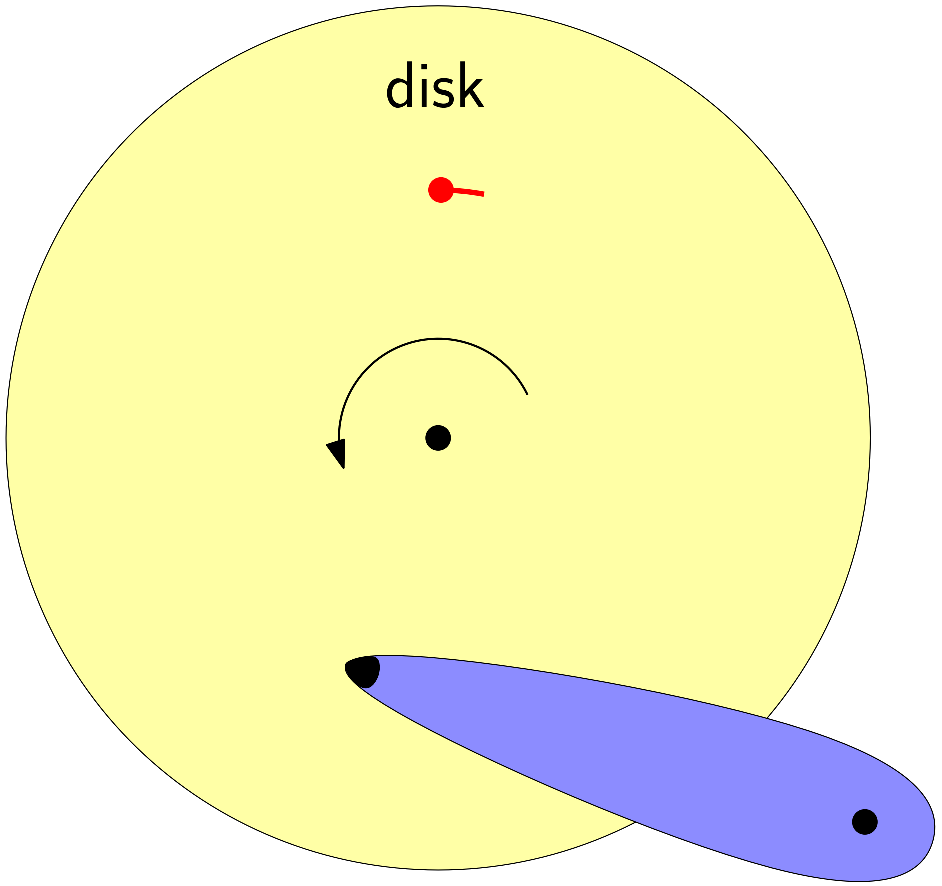

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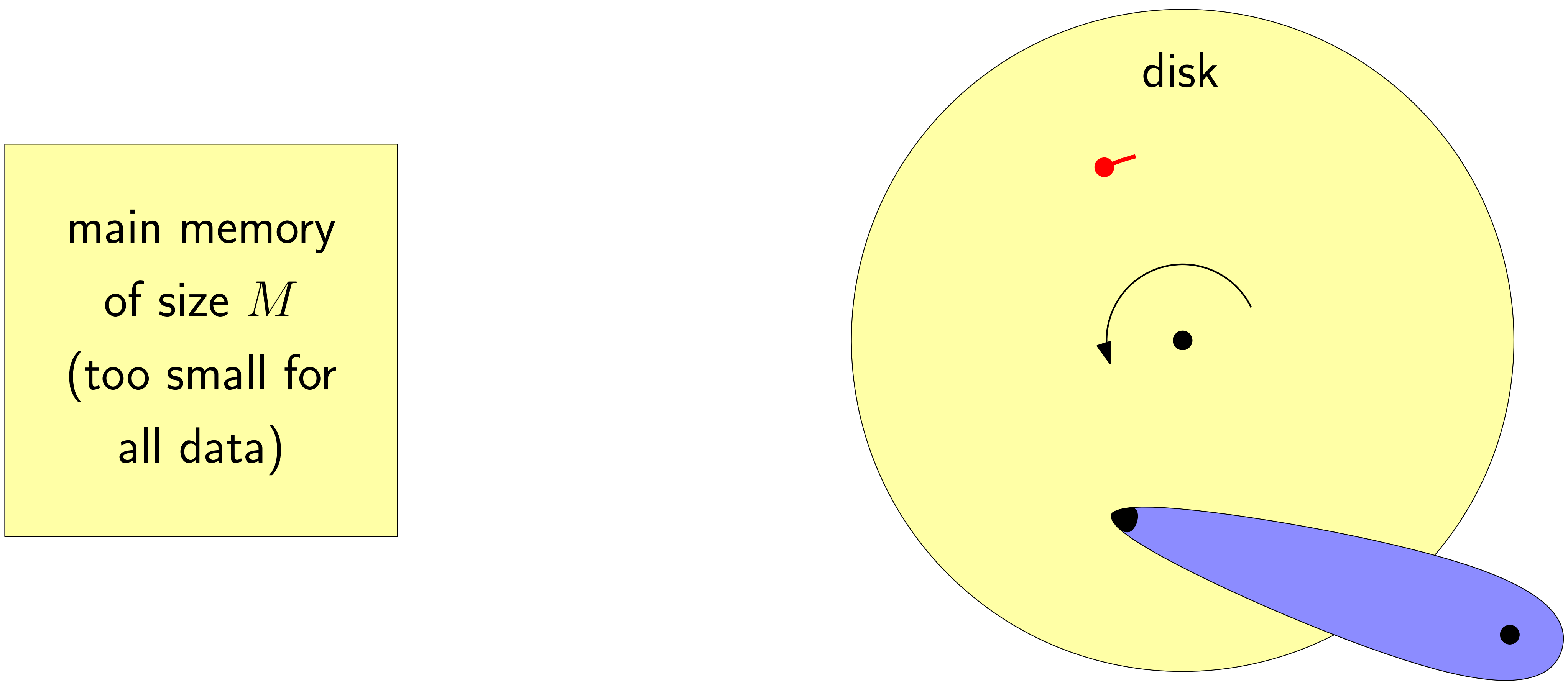

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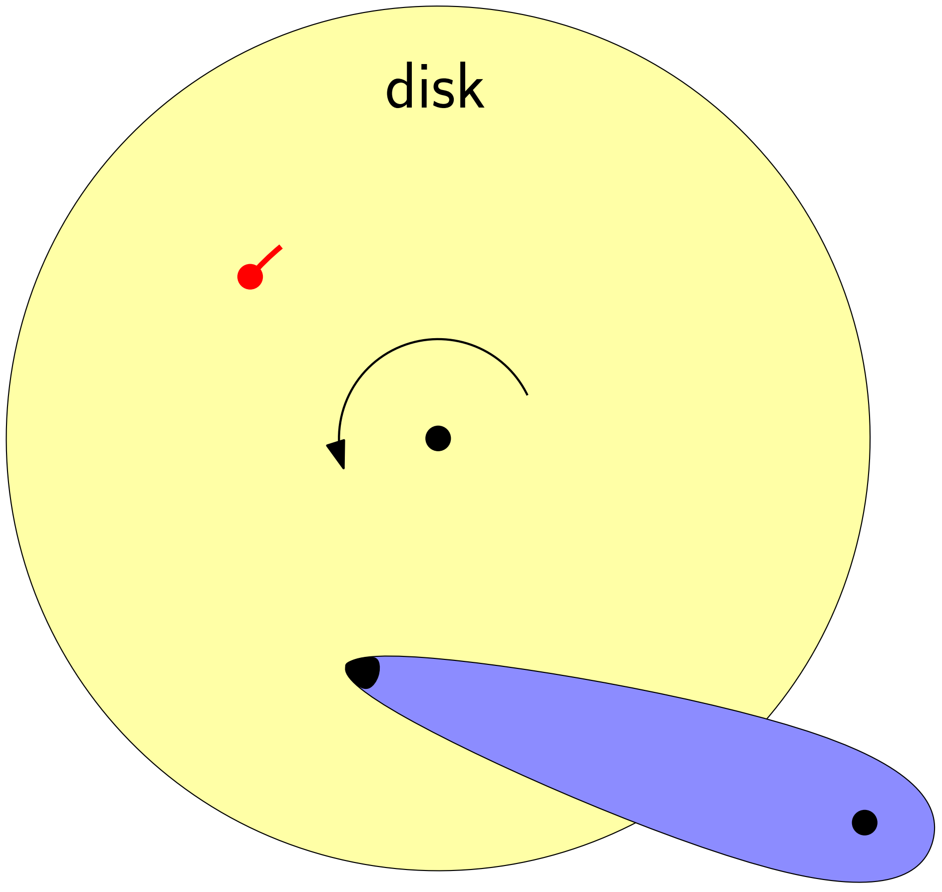

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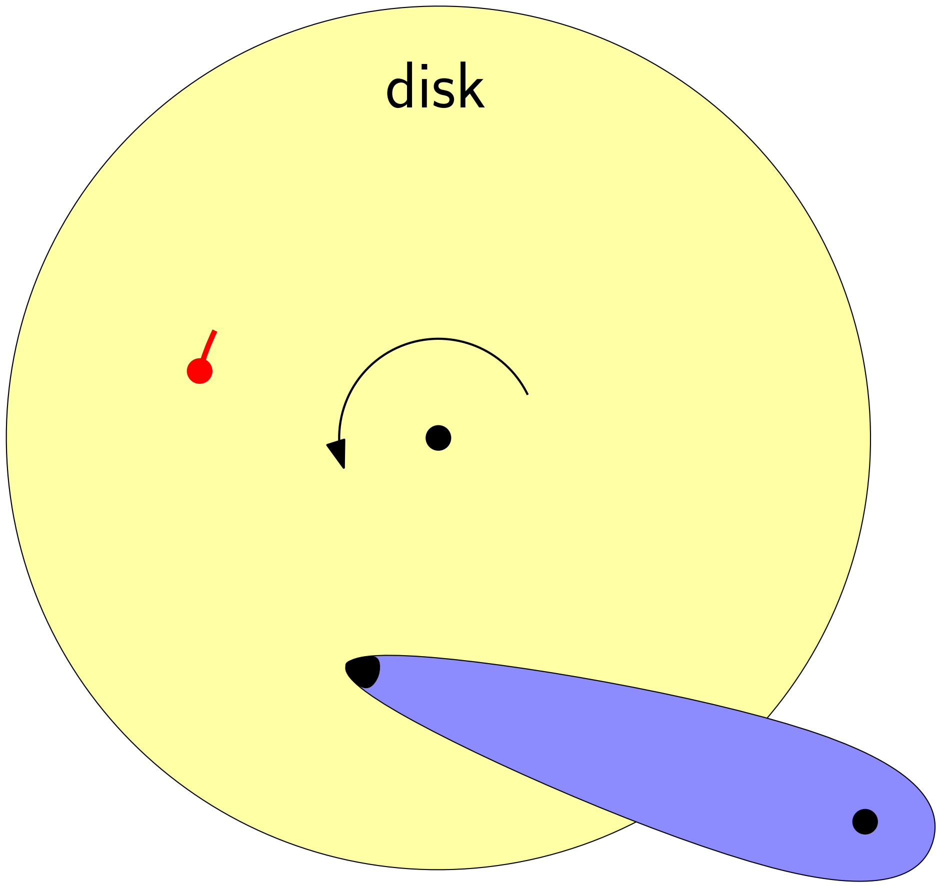

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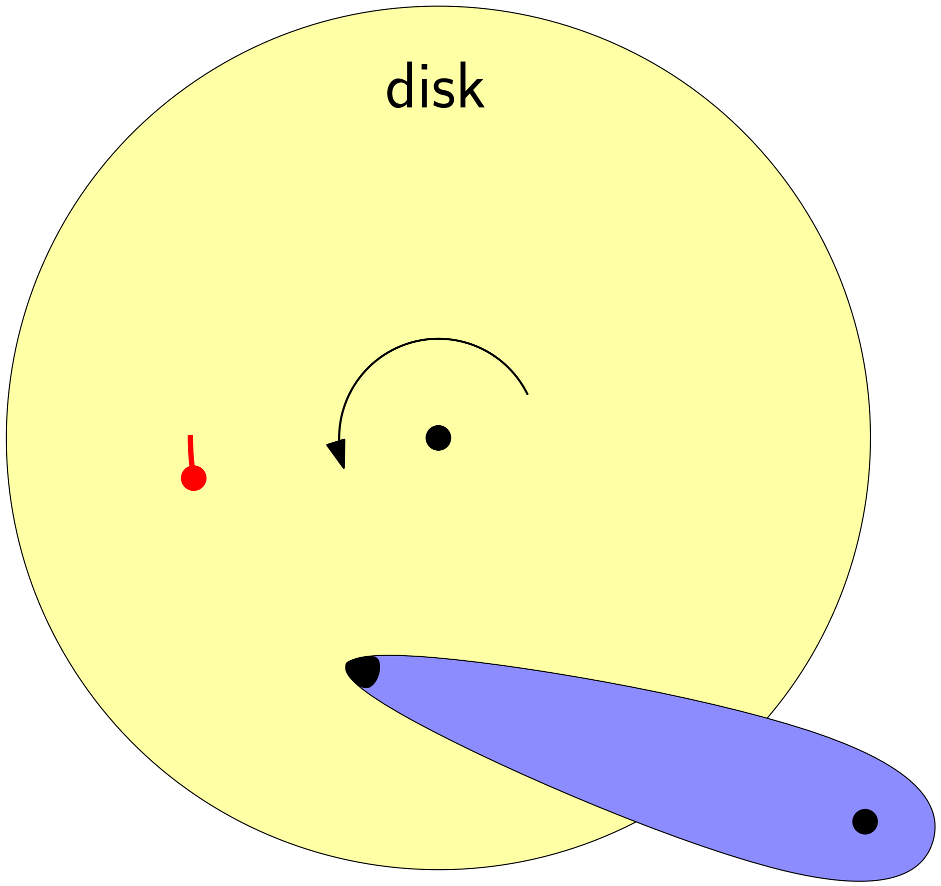

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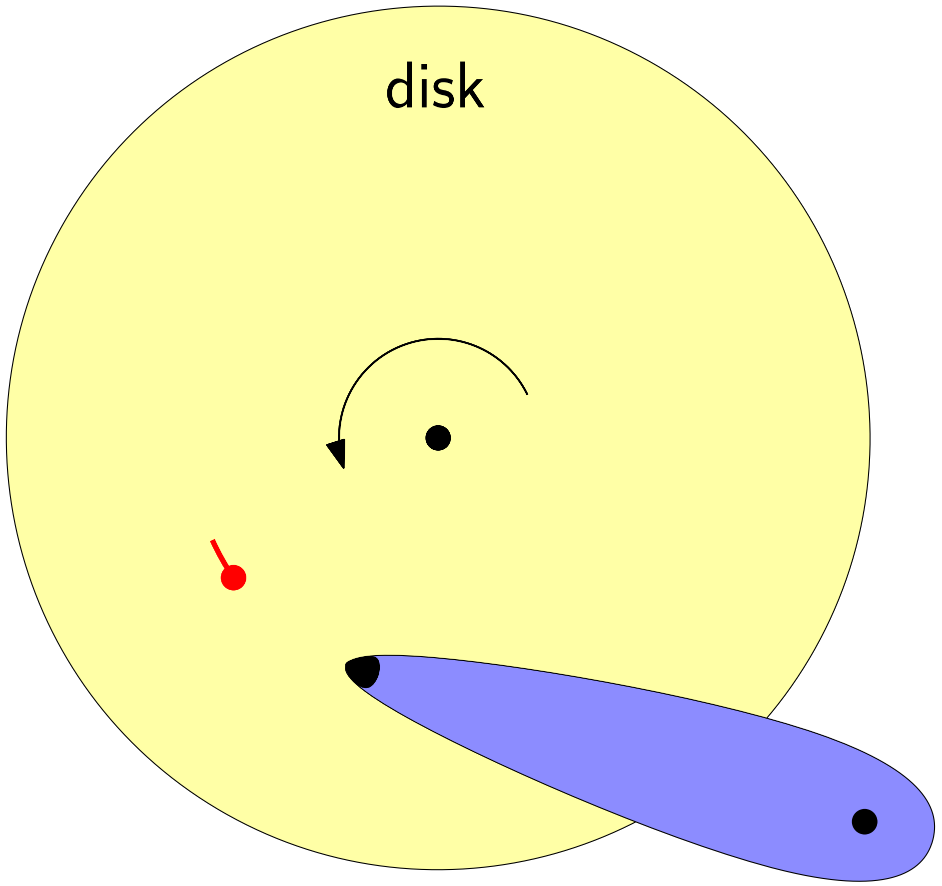

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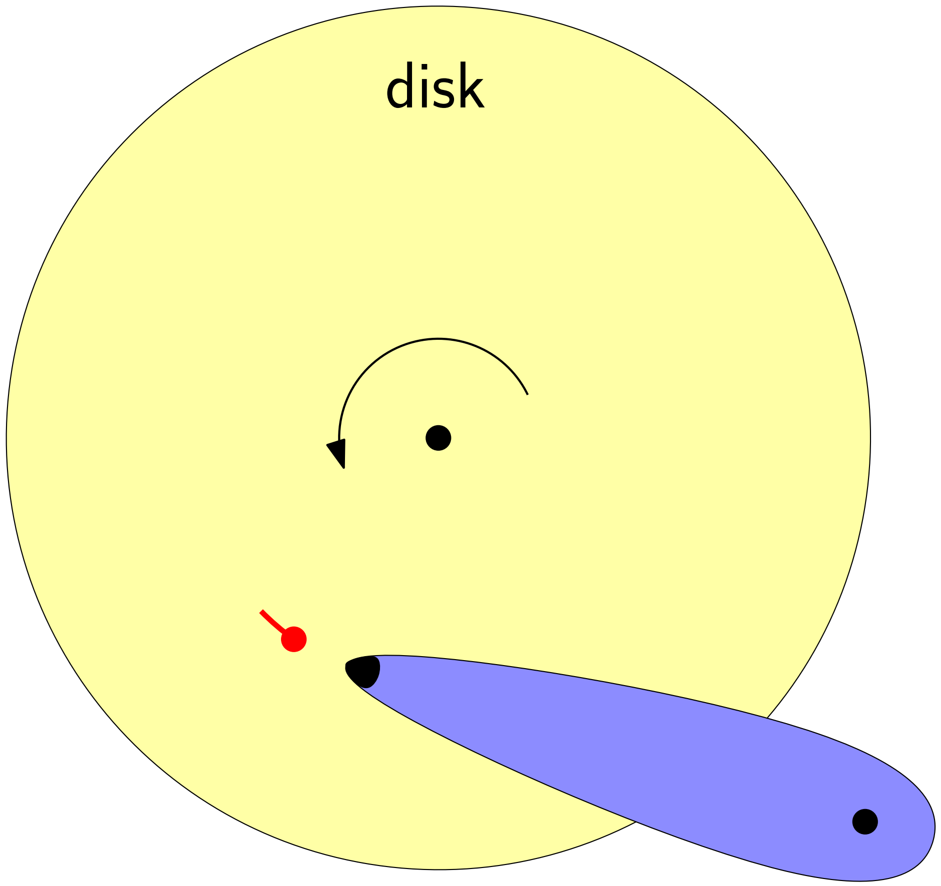
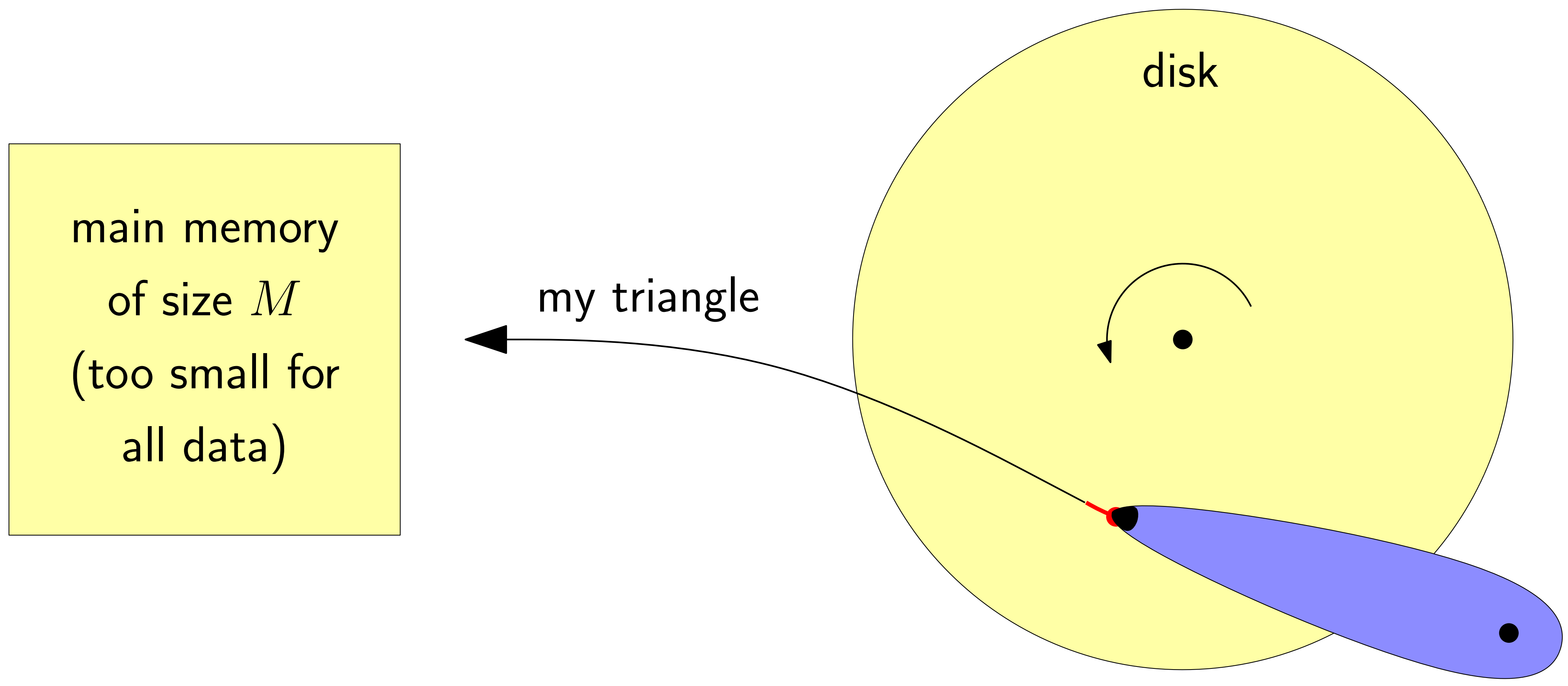


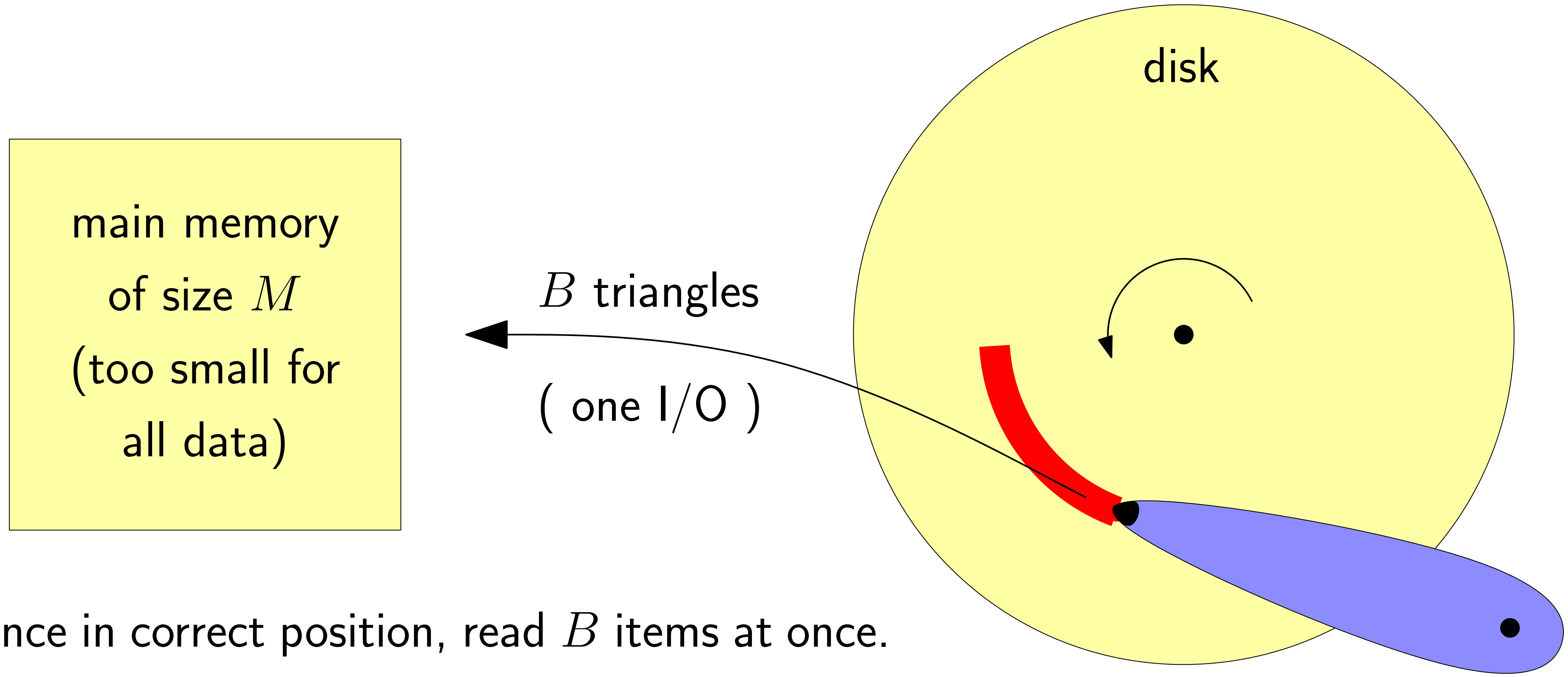
Using external memory

Waiting for one triangle takes ≈ 1 000 000 CPU cycles



Using external memory

Waiting for one triangle takes $\approx 1\,000\,000$ CPU cycles



Solution: once in correct position, read B items at once.
(hope you can keep them in memory until you need them)

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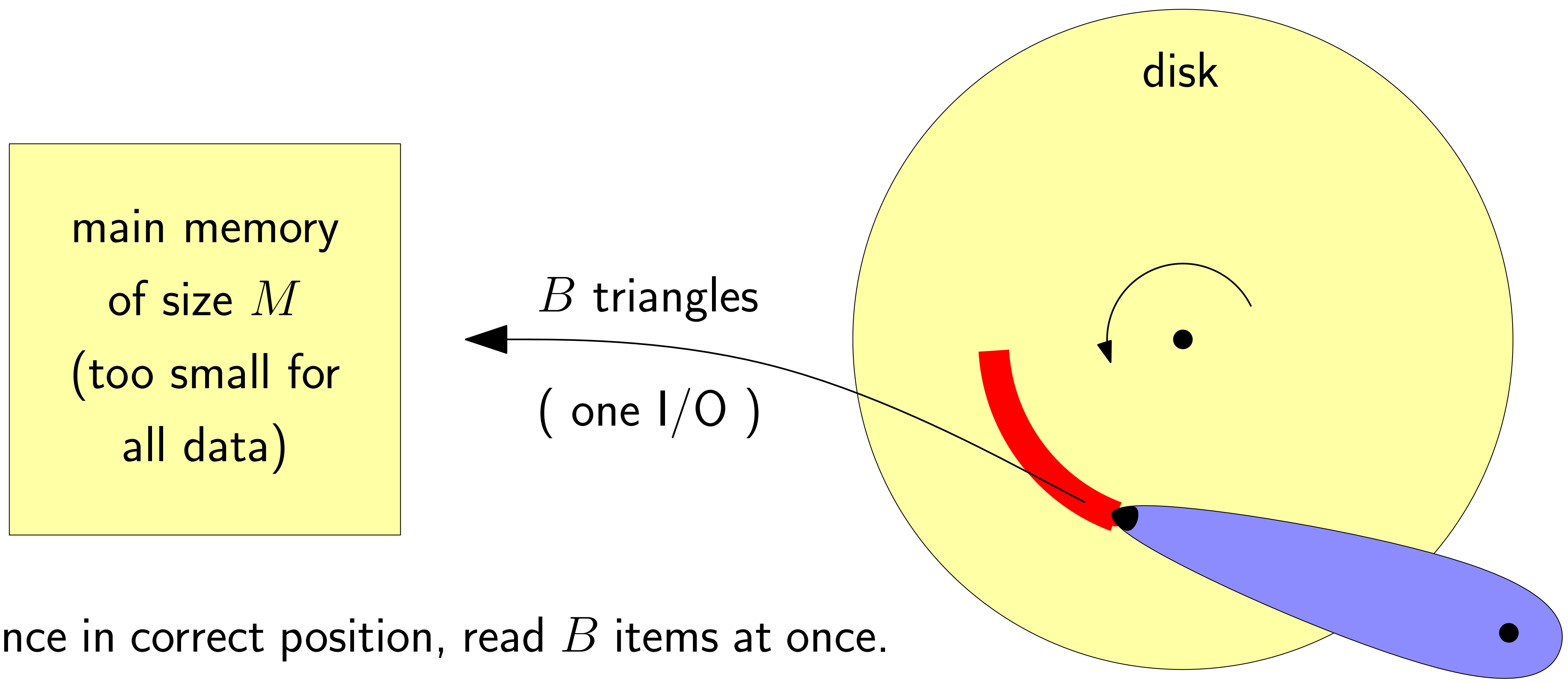

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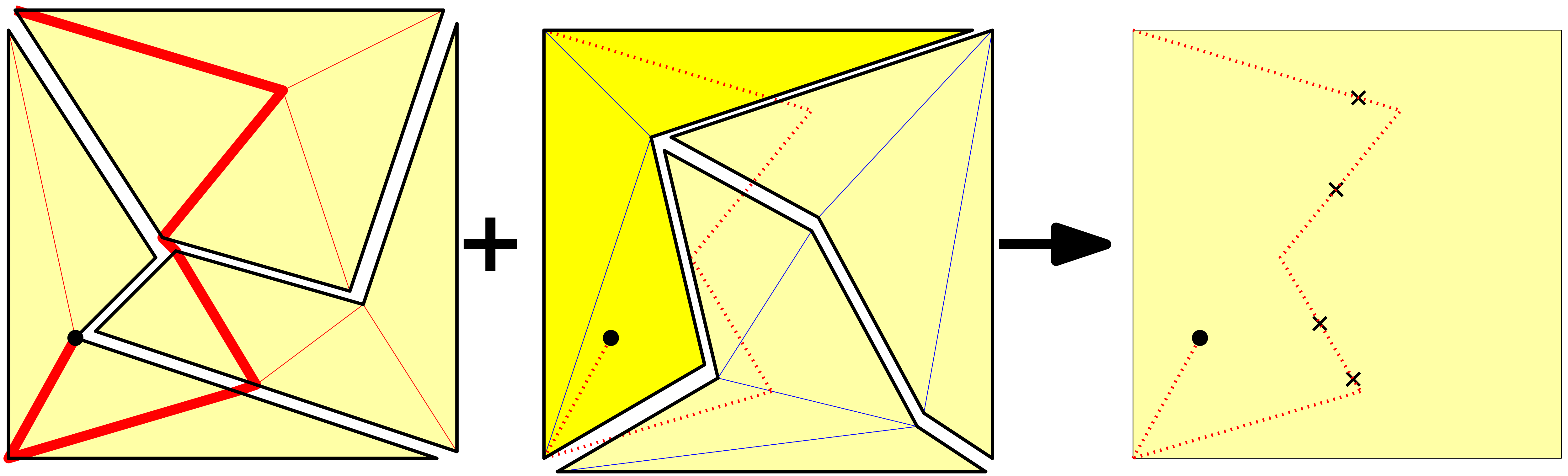
Analysing algorithms that work on data on disk: number of I/O's dominate.

$$scan(n) = \frac{n}{B} \quad < \quad sort(n) = \frac{n}{B} \log_{M/B} \frac{n}{B} \quad << \quad n \quad \text{I/O's}$$

Overlaying triangulations on disk?

Maps: ..., triangulations



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- $\Theta(1)$ operations per edge
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Total: $\Theta(n + k)$ CPU-operations (for n triangles, k intersections)

On disk, data arranged in blocks.

Overlaying triangulations on disk?

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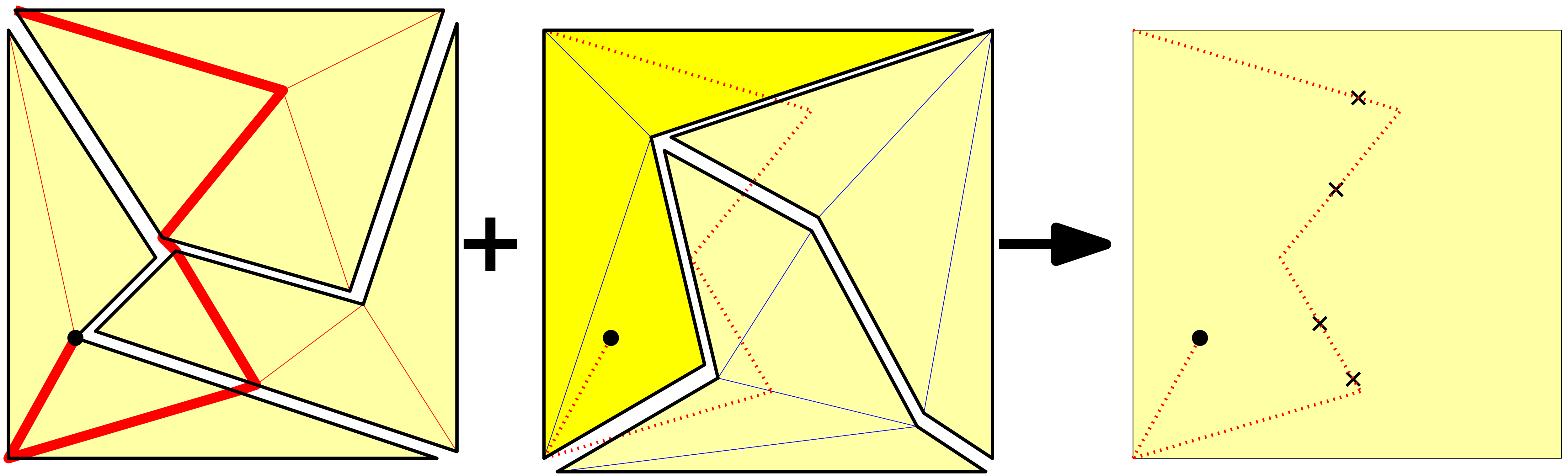

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Total: $\Theta(n+k)$ CPU-operations (for n triangles, k intersections)

On disk, data arranged in blocks. 1 I/O $\approx$ 1,000,000 CPU-ops. $\Theta(n+k)$ I/O's?

Our results

n = input size;
M = main memory size;
B = disk block size

$$scan(n) = \frac{n}{B} \quad < \quad sort(n) = \frac{n}{B}\log_{M/B}\frac{n}{B} \quad << \quad n$$

Previously:

- Arge et al.: map overlay in $O(sort(n) + k/B)$ I/O's (complicated, super-linear space)
- Crauser et al.: randomized, linear space

Our results: in $O(sort(n))$ I/O's we can build a data structure that supports:

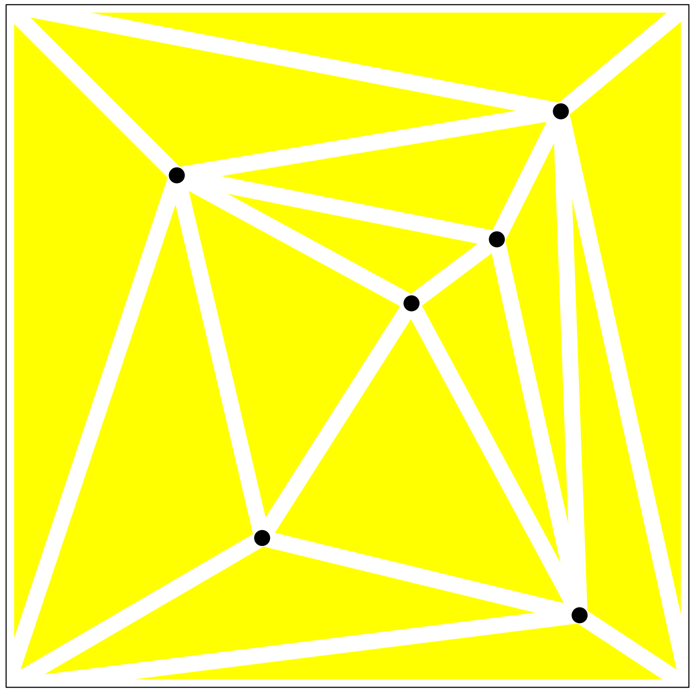
- map overlay in $O(scan(n))$ I/O's;
- point location in $O(\log_B n)$ I/O's;
- range queries in $O(\frac{1}{\varepsilon}(\log_B n) + scan(k_\varepsilon))$ I/O's;
- for triangulations: basic updates in $O(\log_B n)$ I/O's.

Condition: input must be *fat* triangulation (all angles $>$ positive constant), or a *low-density* set of segments (for any circle C, #intersecting segments $> \mathrm{diam}(C)$ is $O(1)$)

Ingredients: quadtrees ...

Quadtree: divide unit square into quadrants, refine until amount of data per cell is small.

(for example: until every cell has at most one vertex)



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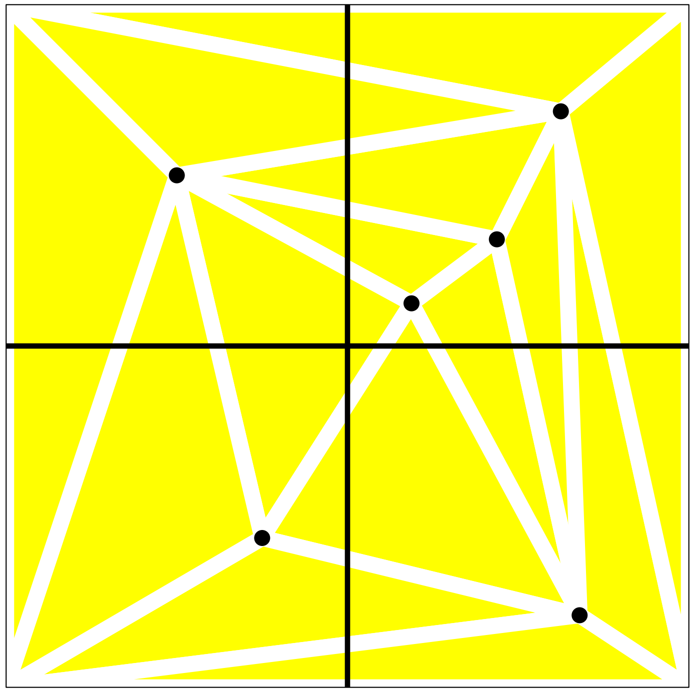

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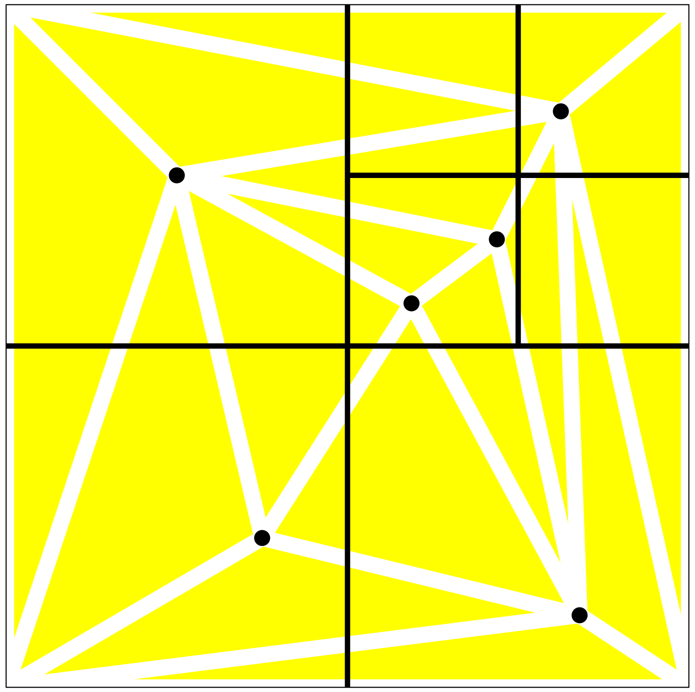

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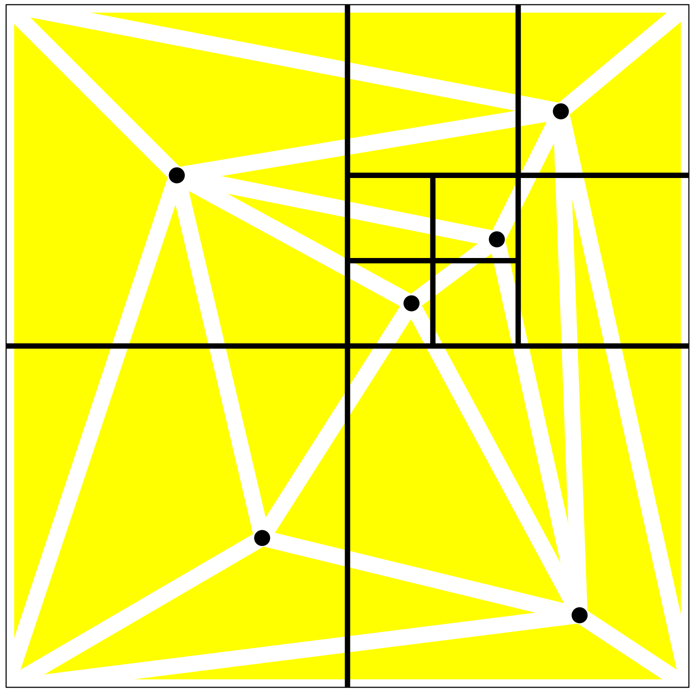
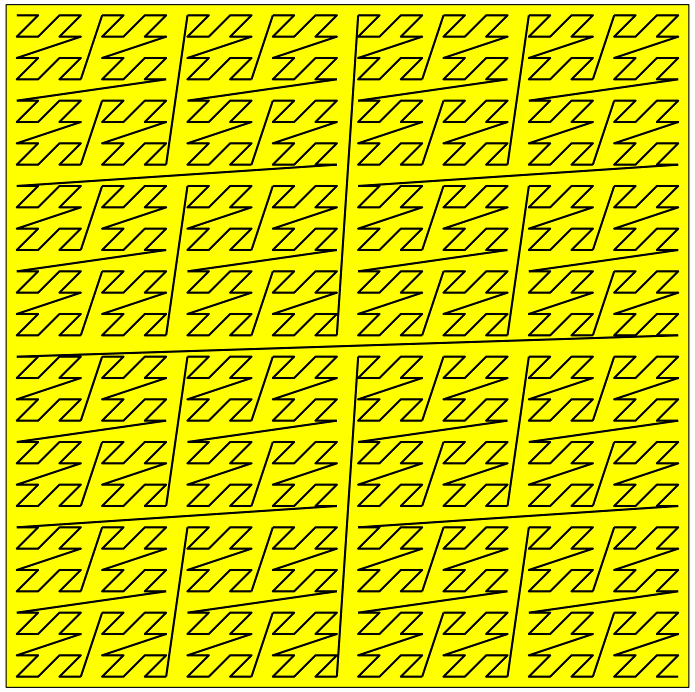


Ingredients: ... and Z-order

Z-order space-filling curve: visit quadrants recursively in order NW, NE, SW, SE



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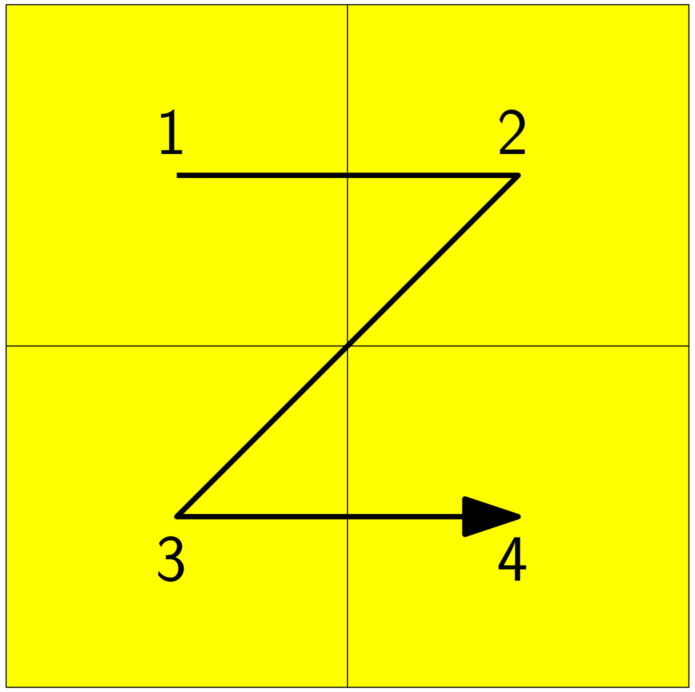

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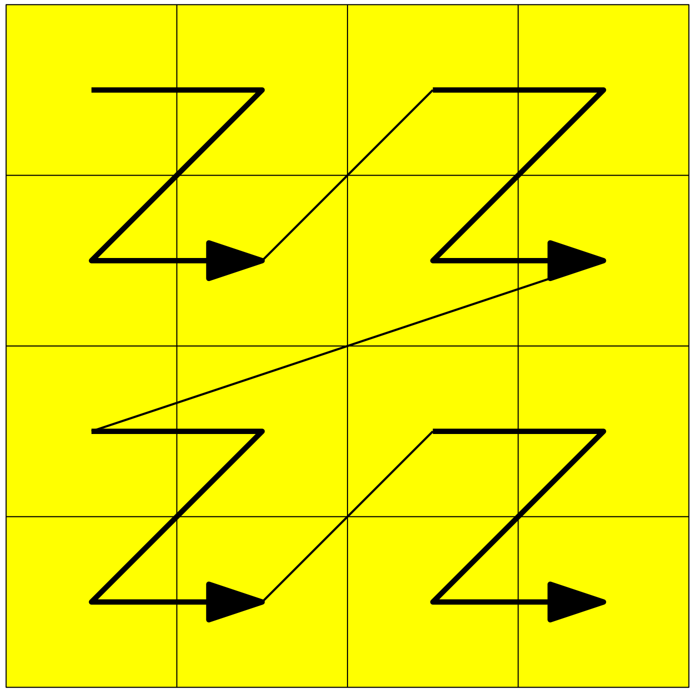

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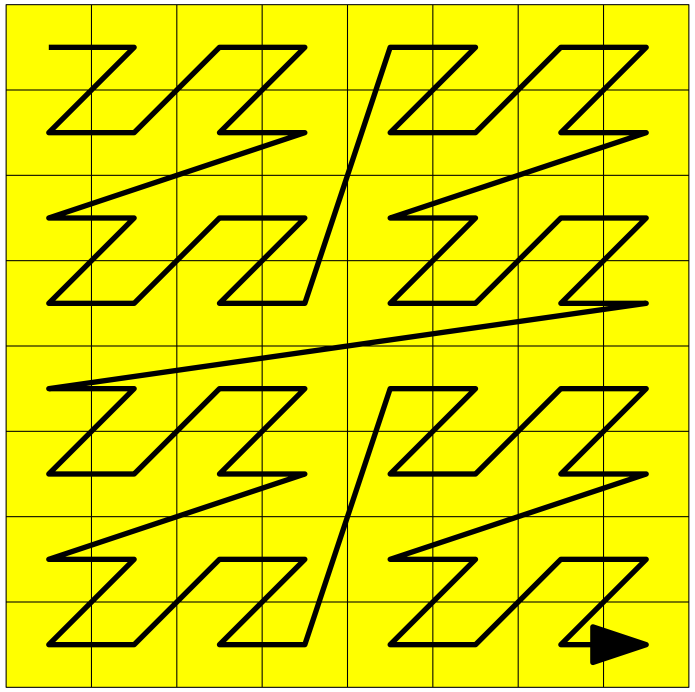

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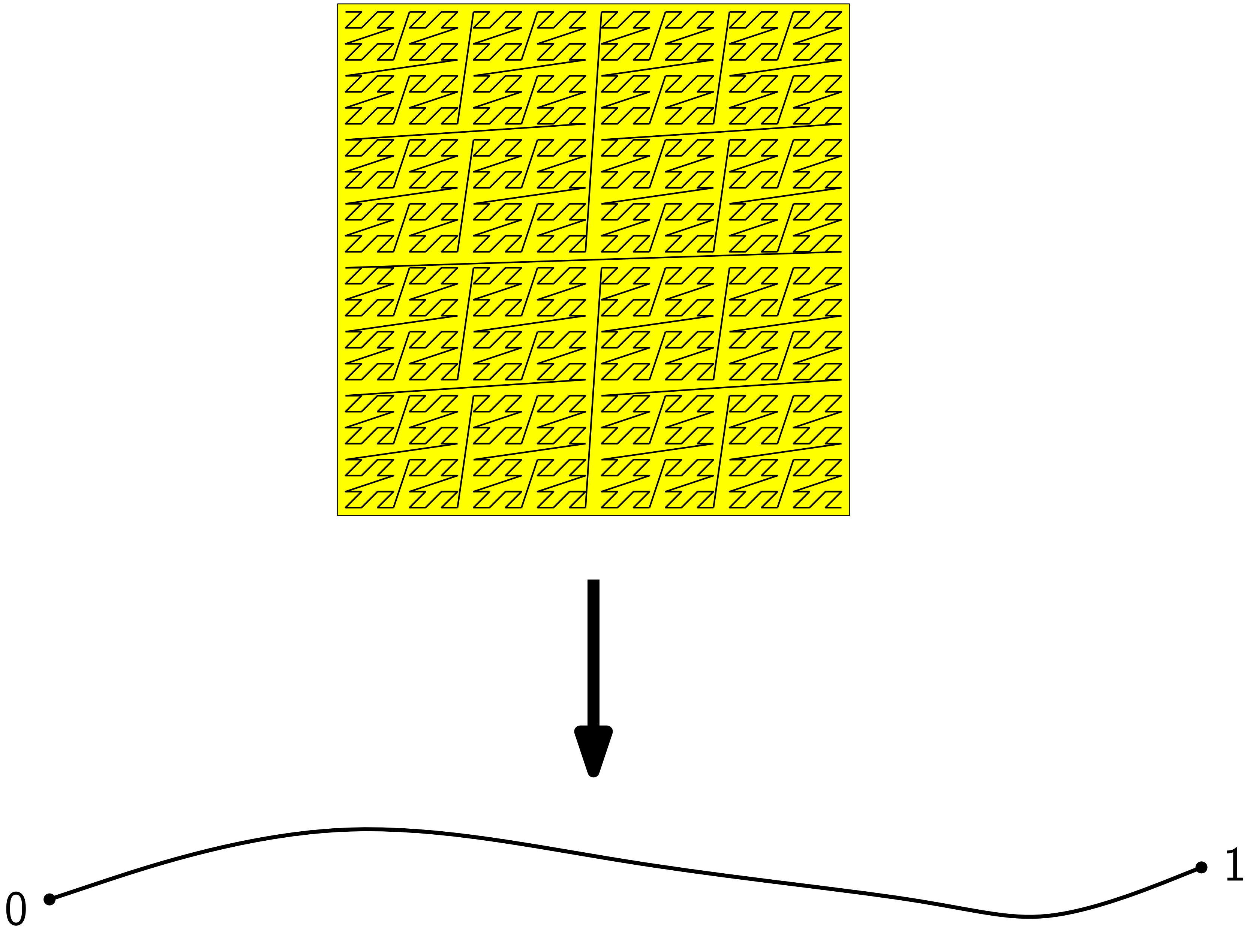
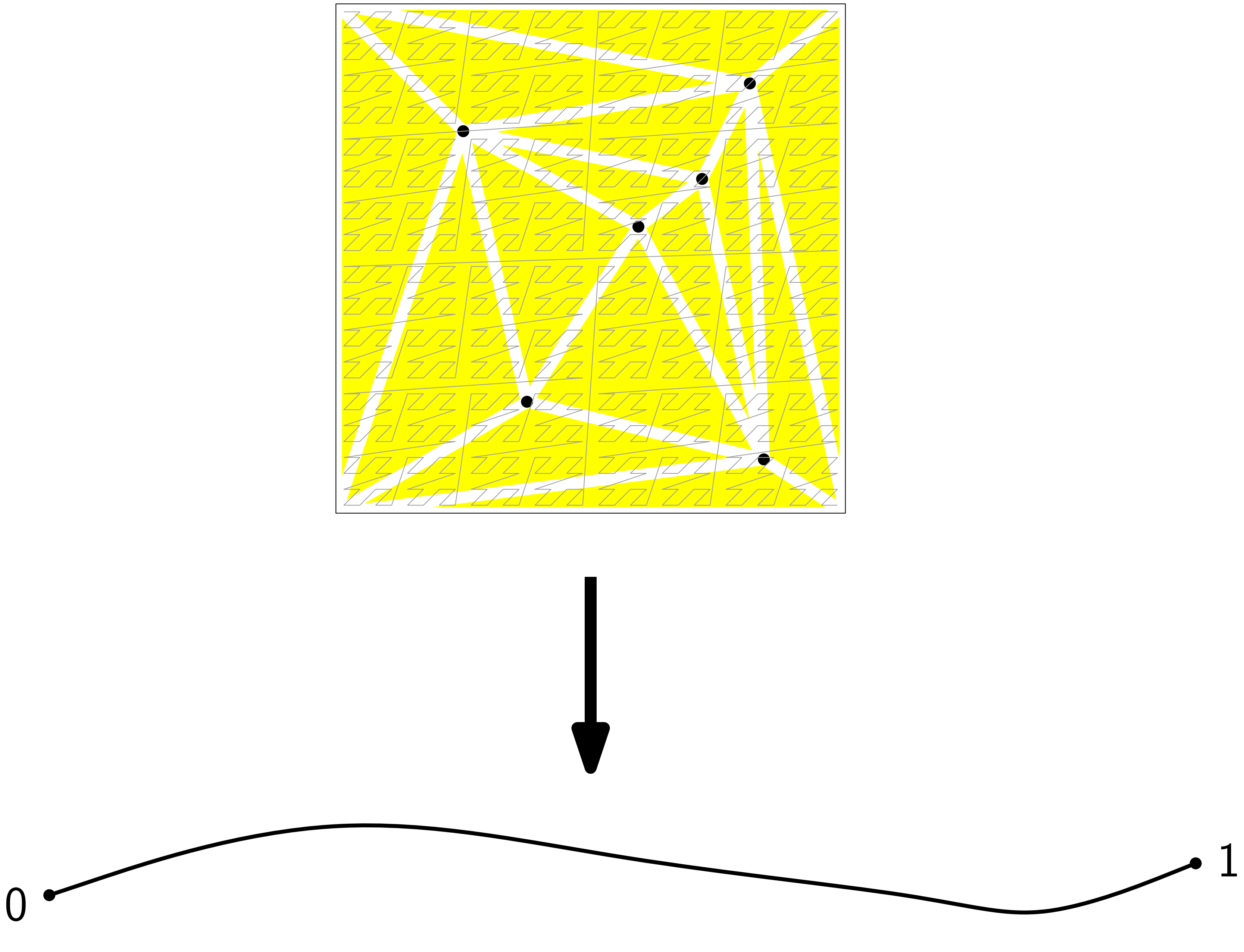


Ingredients: quadtrees and Z-order

Quadtree cell $\equiv$ interval on Z-order curve

Quadtree subdivision $\equiv$ subdivision of Z-order curve



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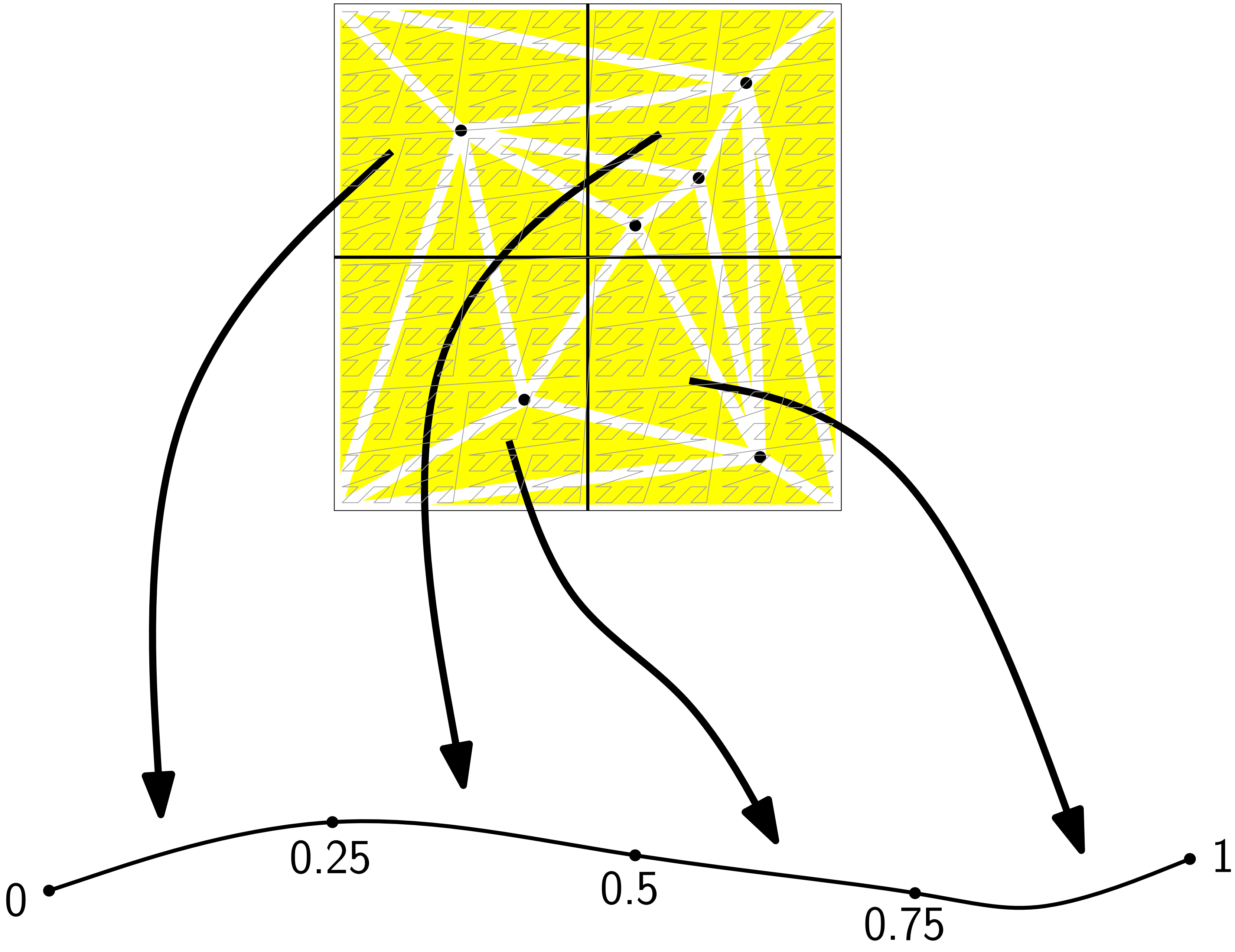

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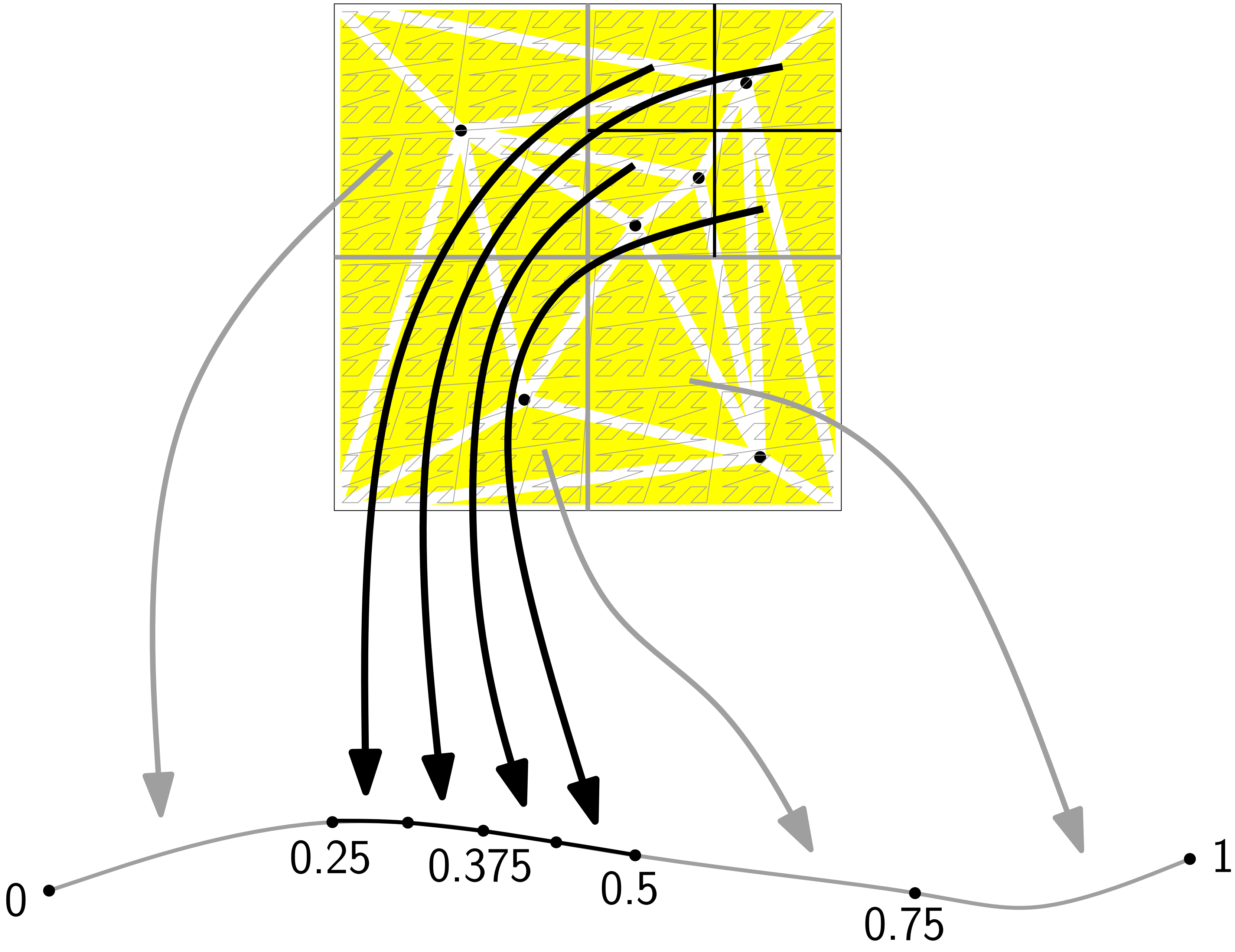

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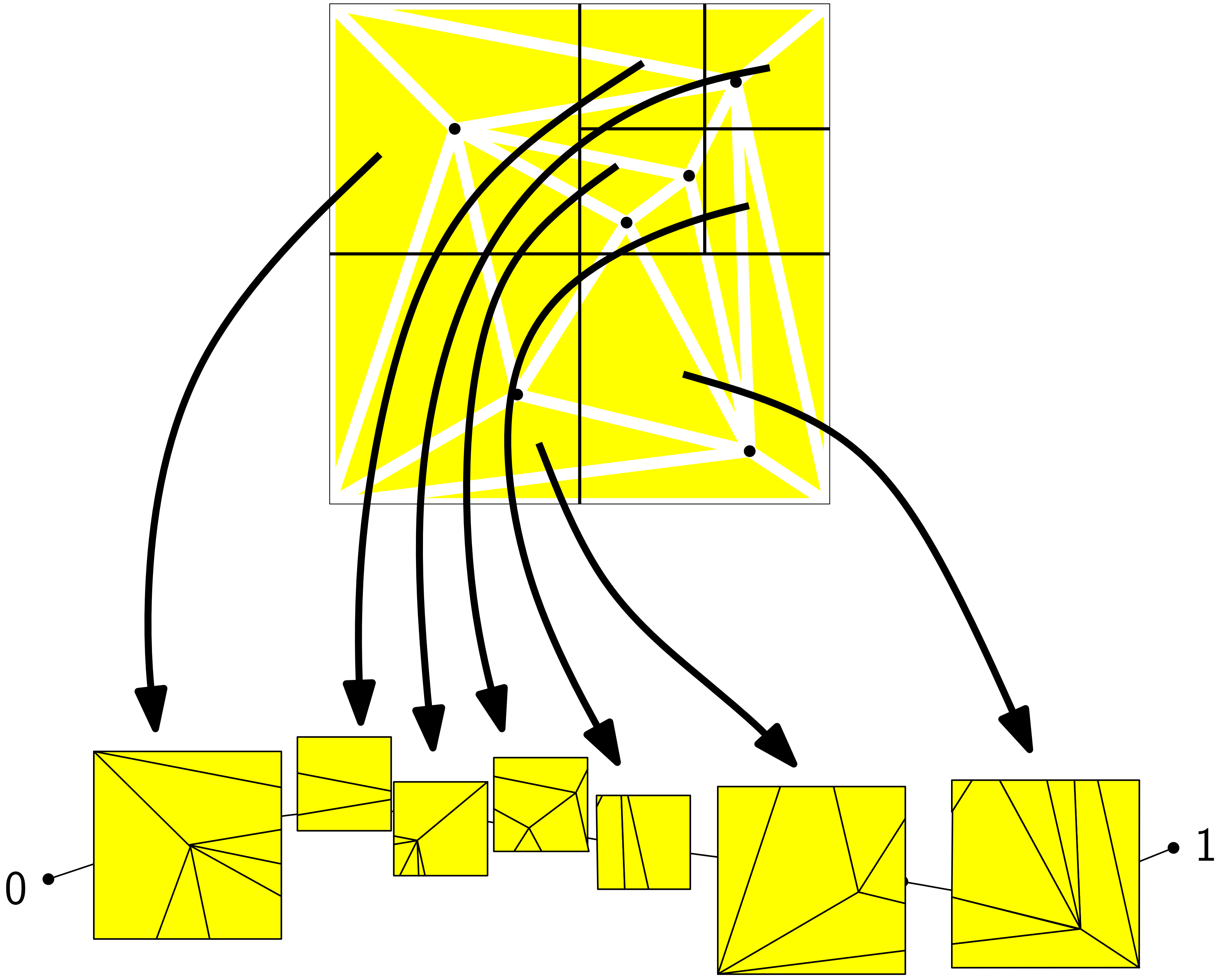
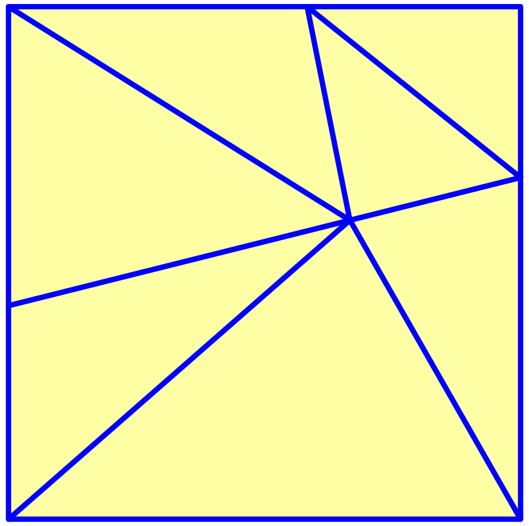
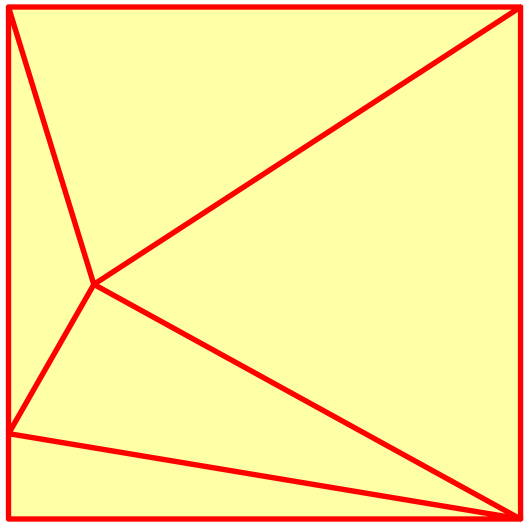


Map overlay with quadtrees in Z-order





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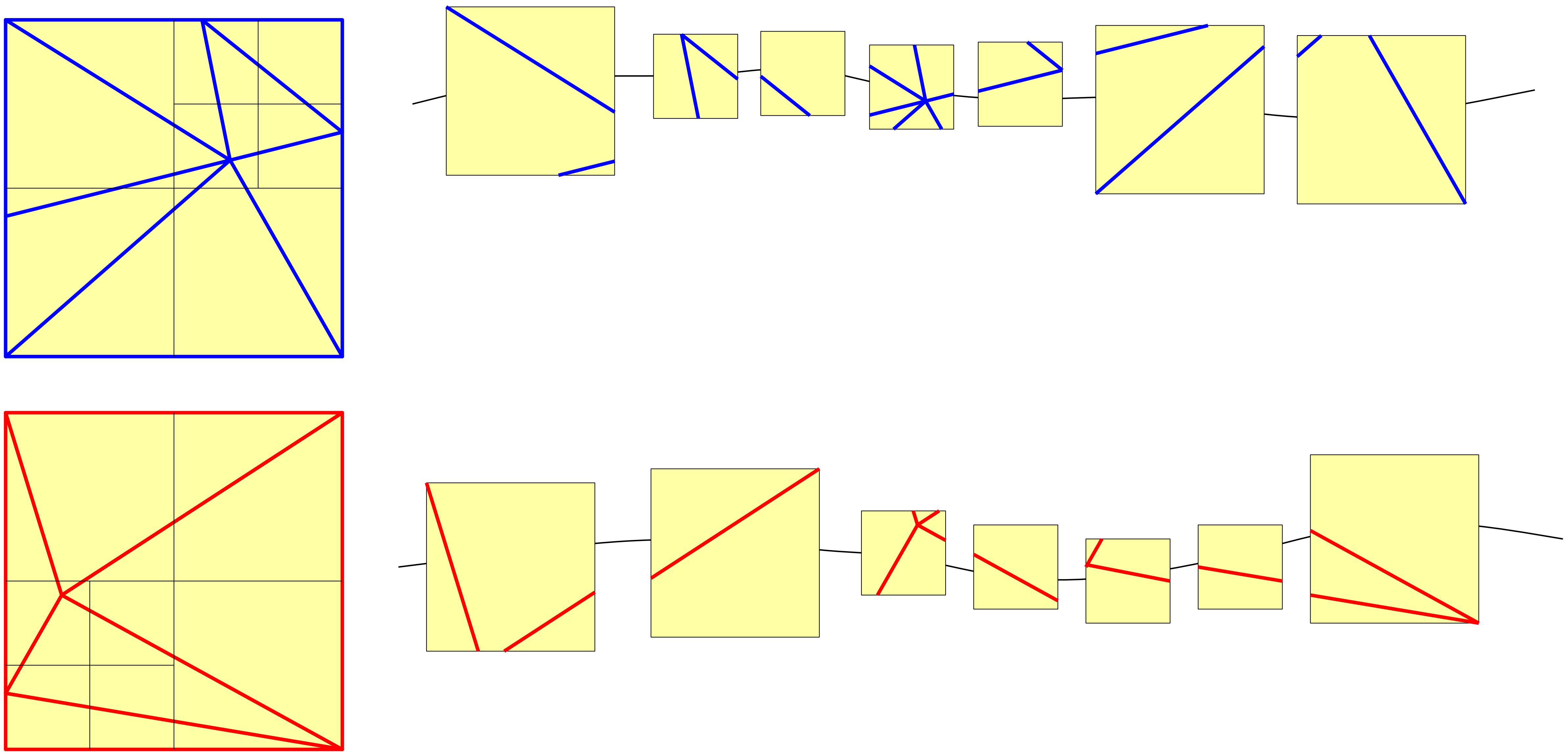

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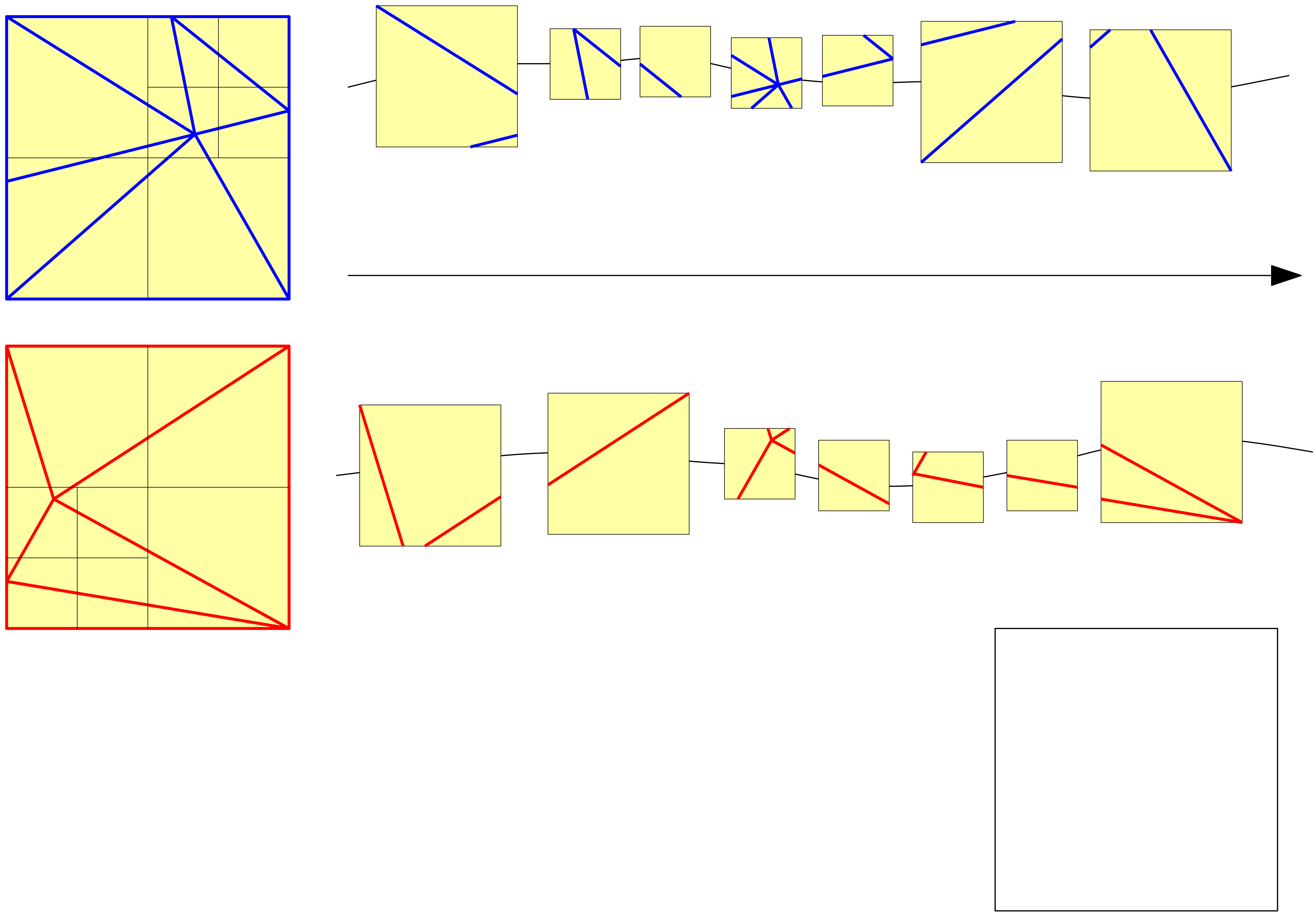

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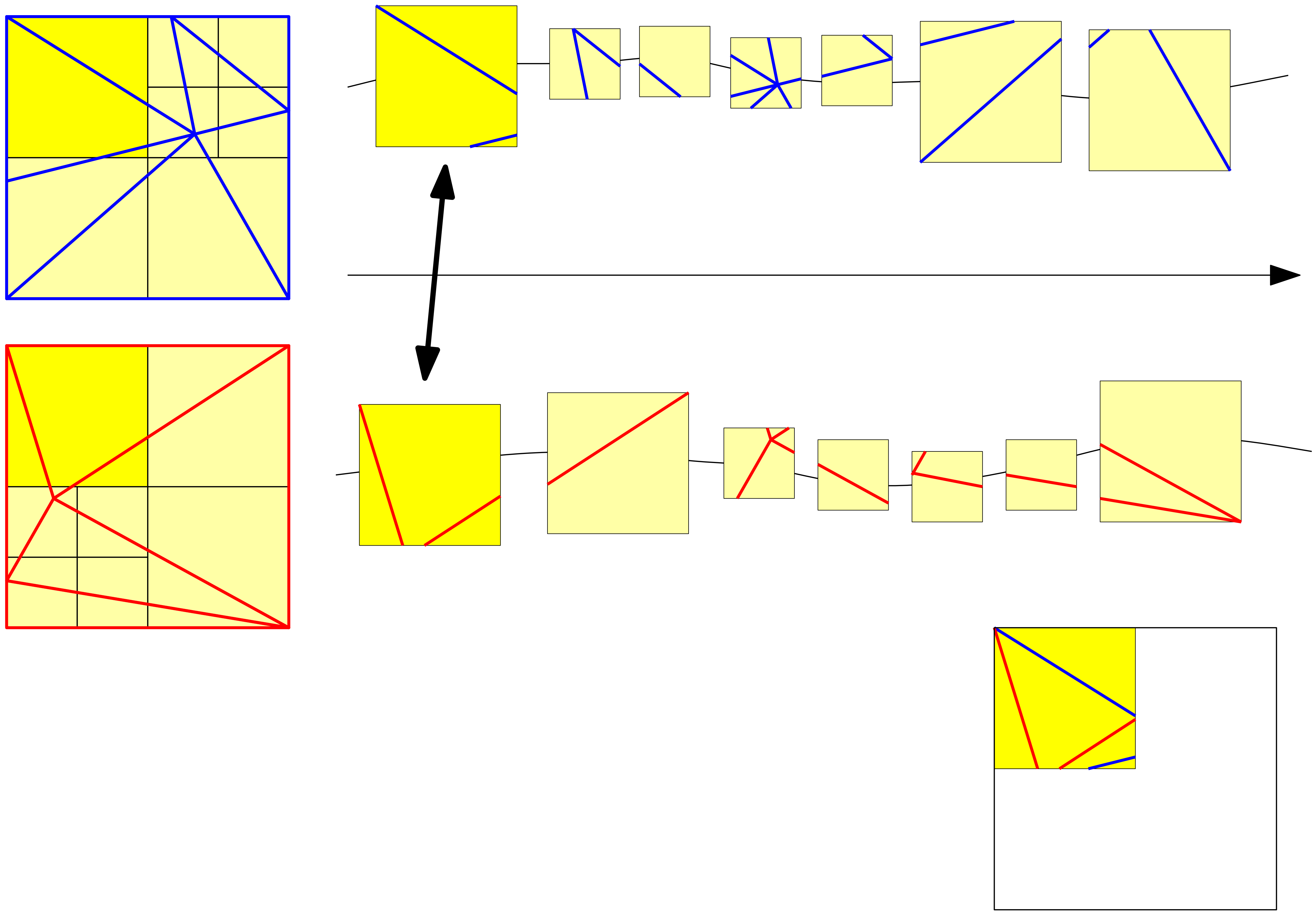

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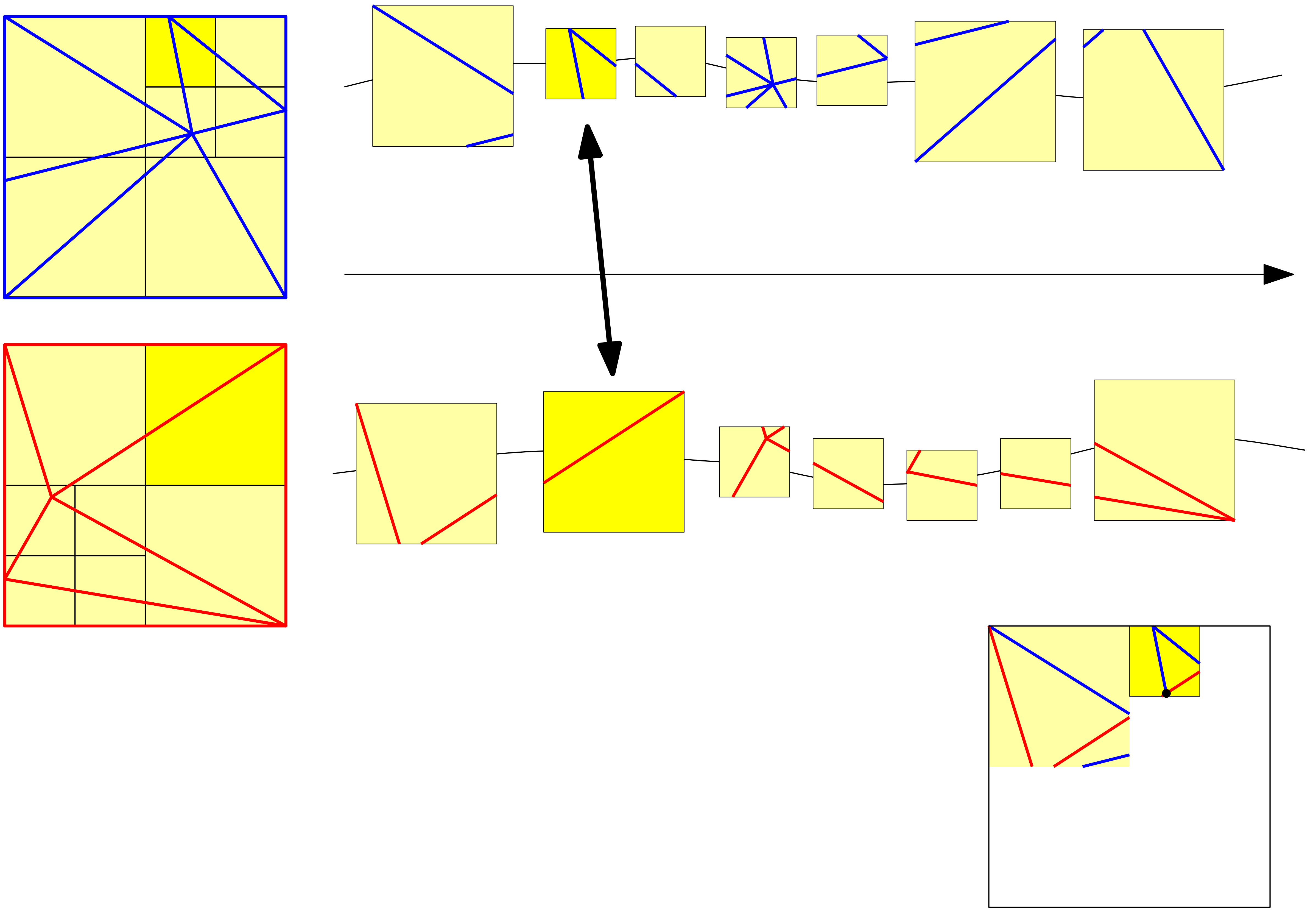

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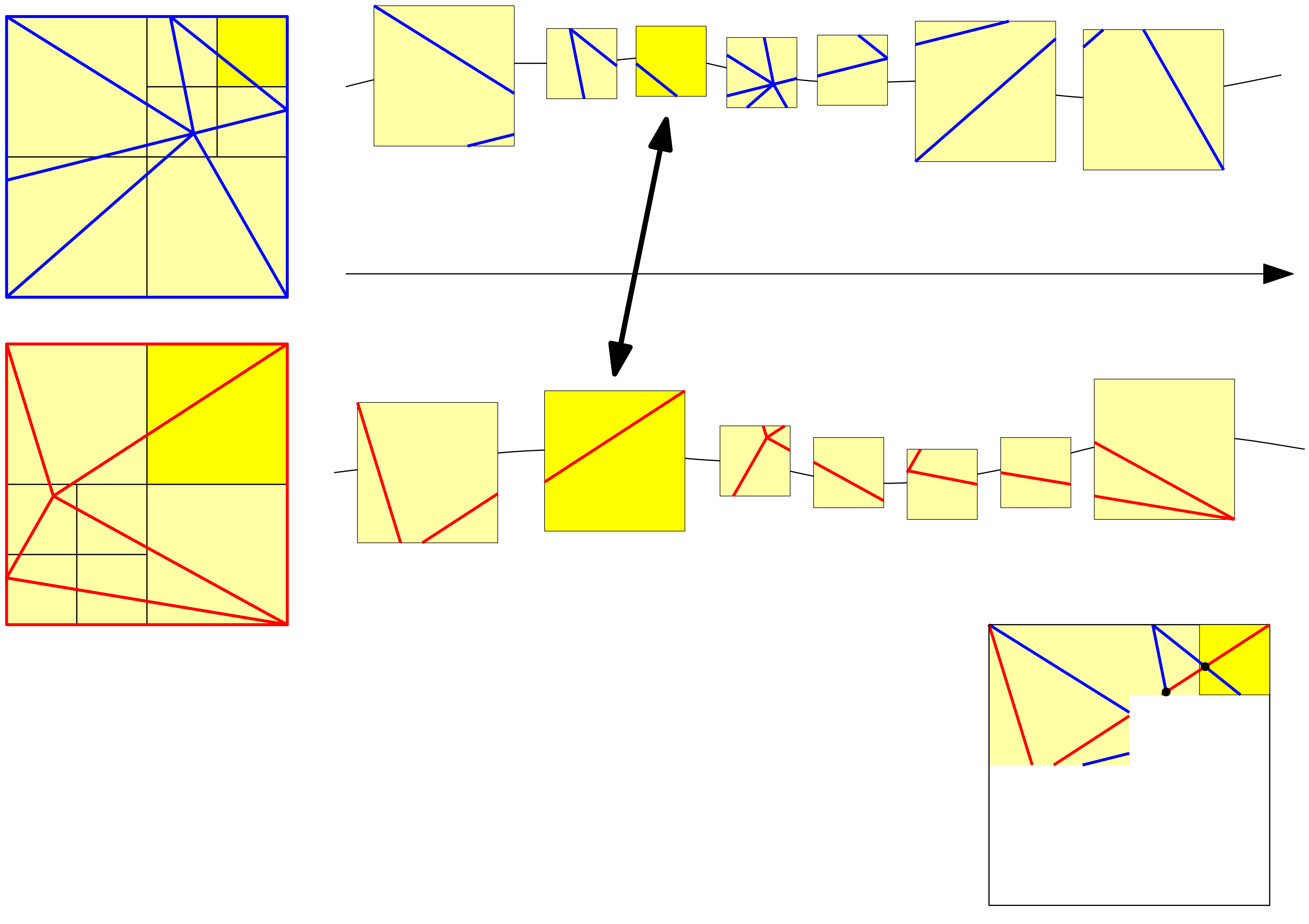

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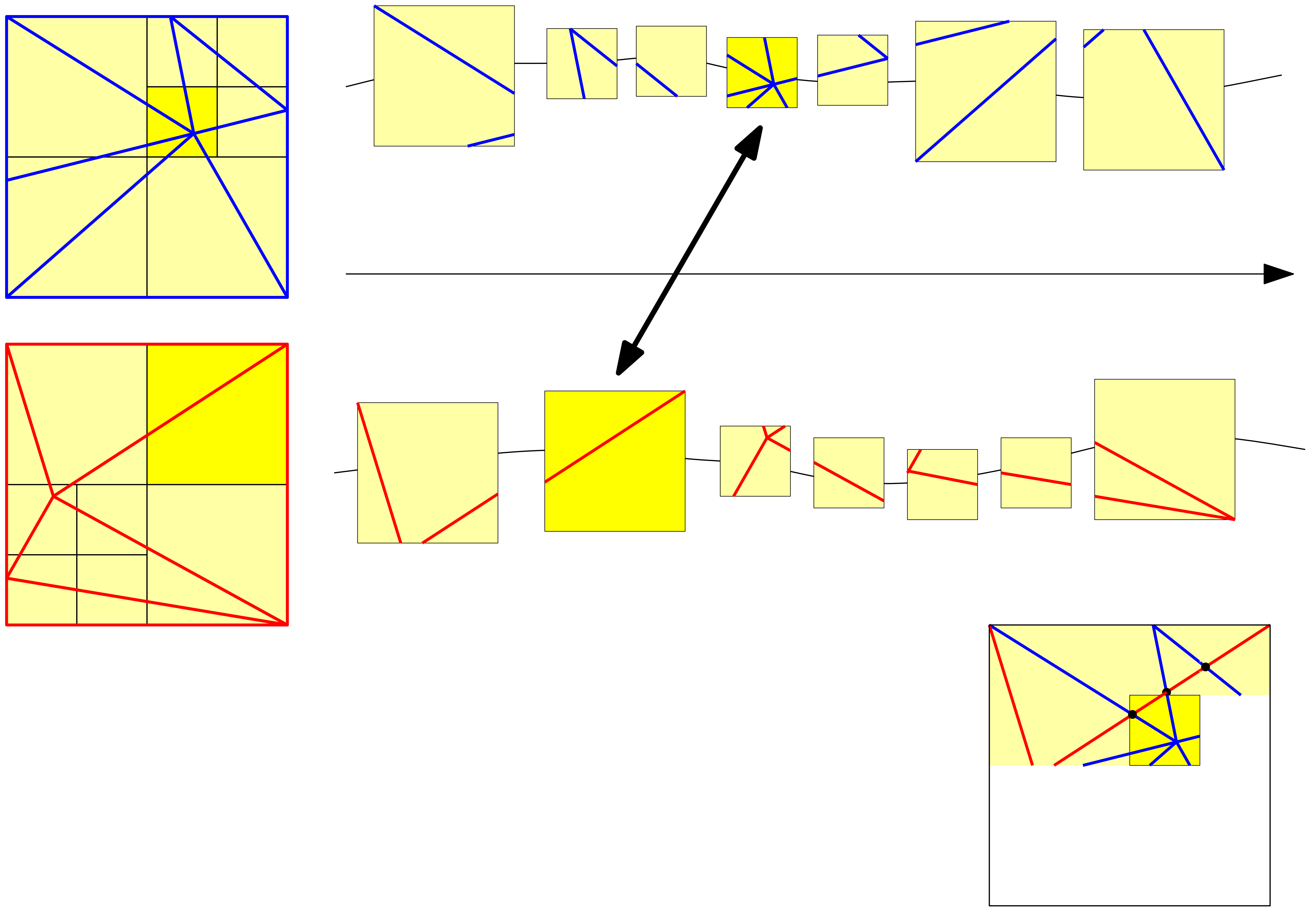
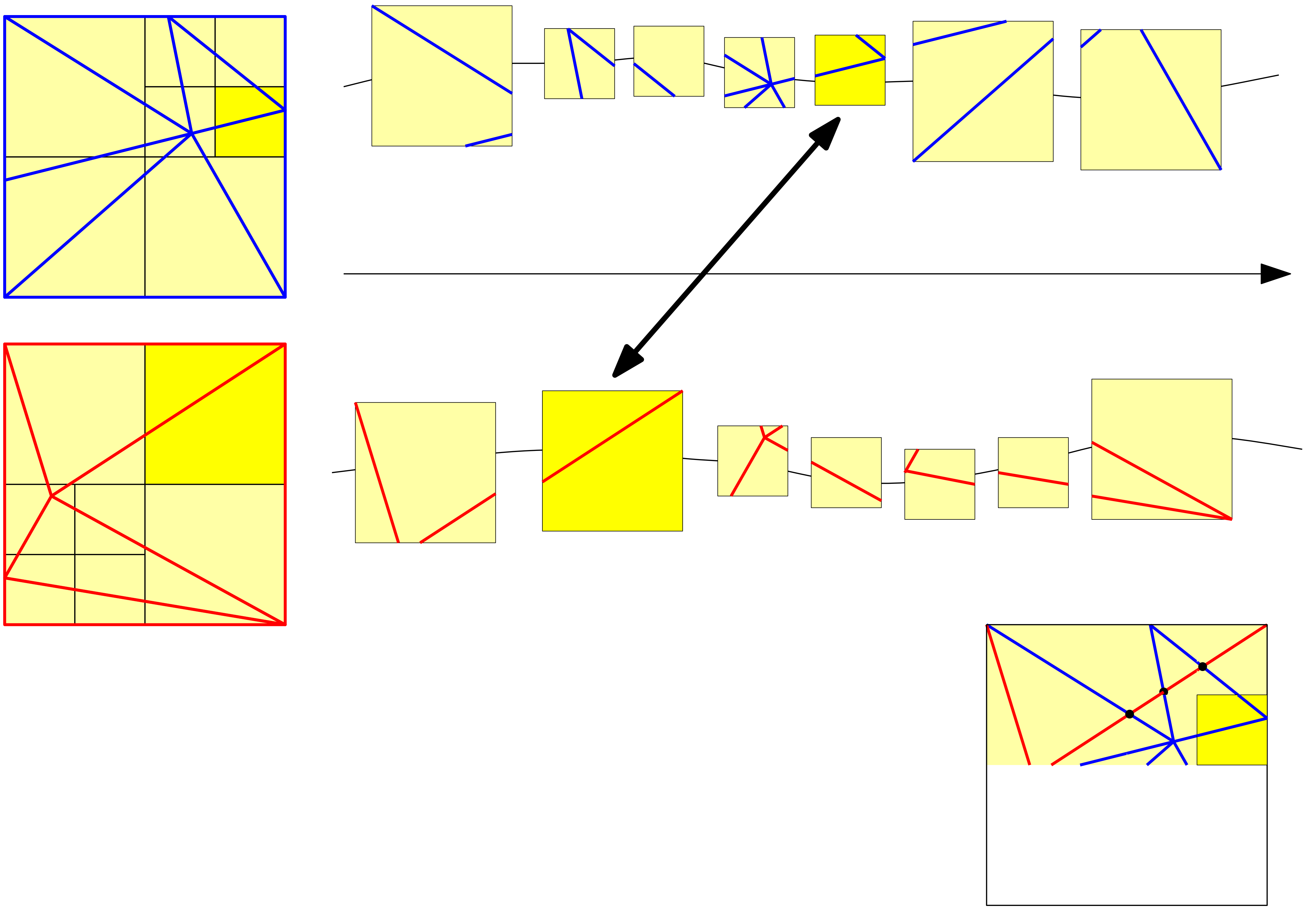


Map overlay with quadtrees in Z-order



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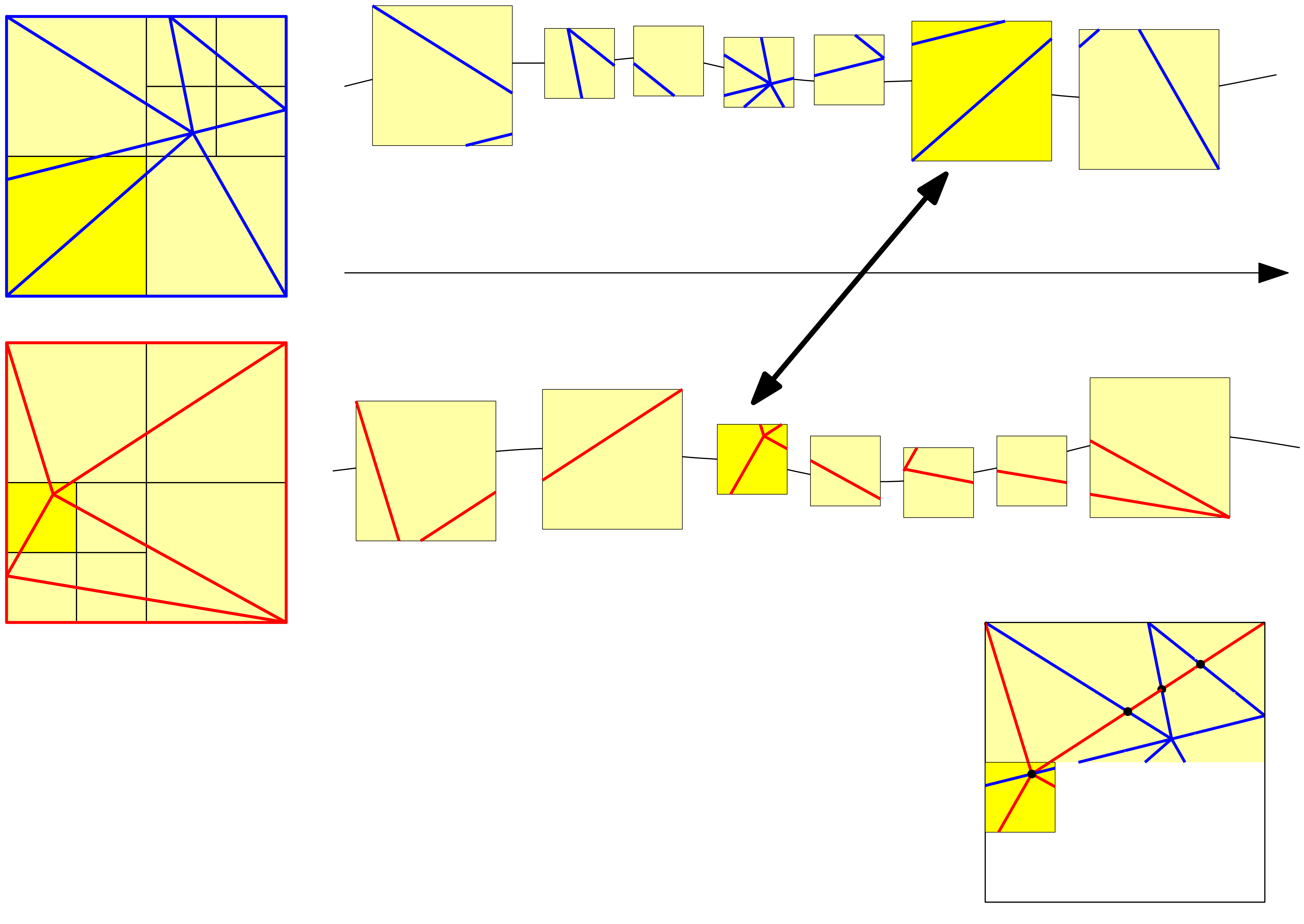

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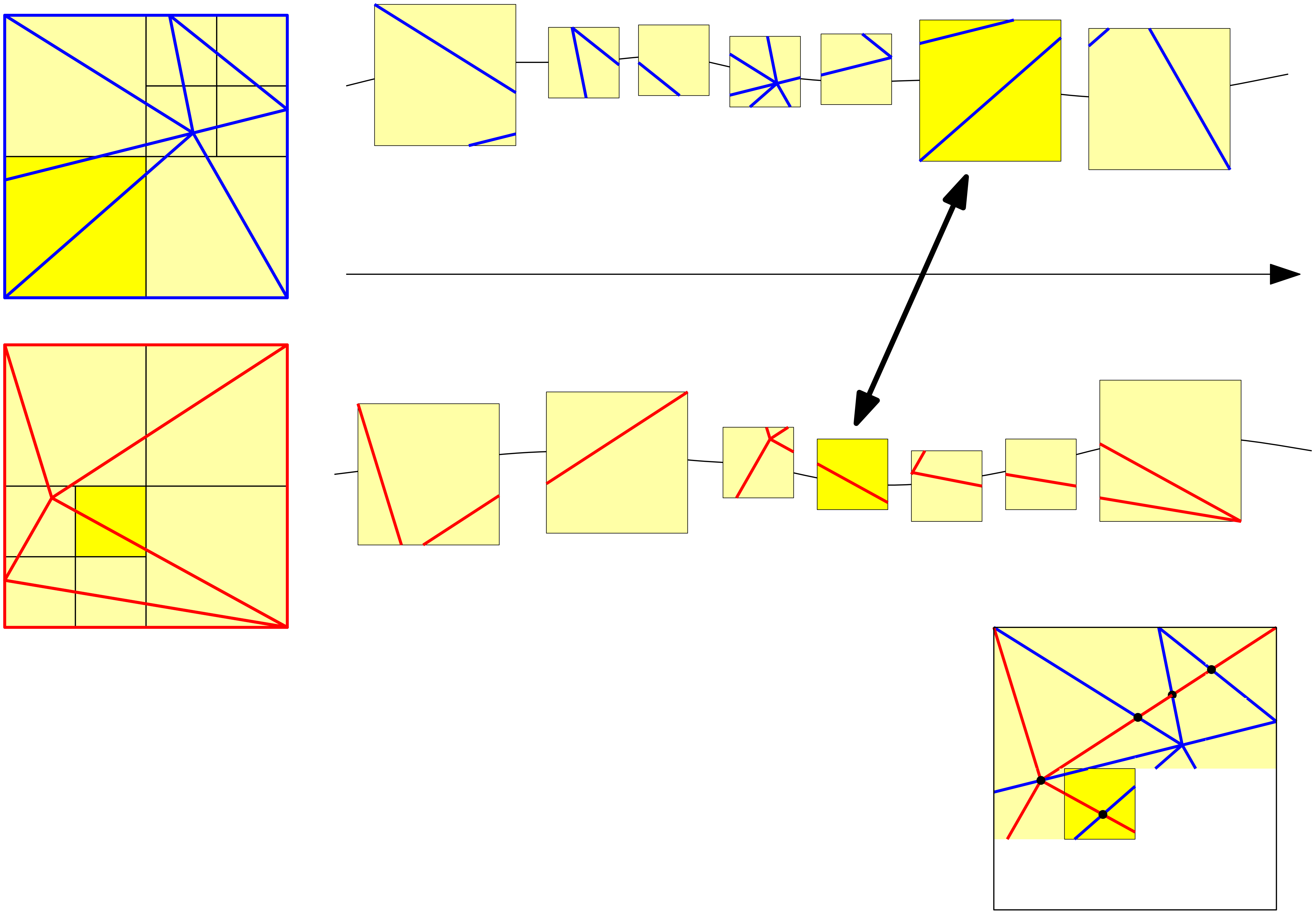

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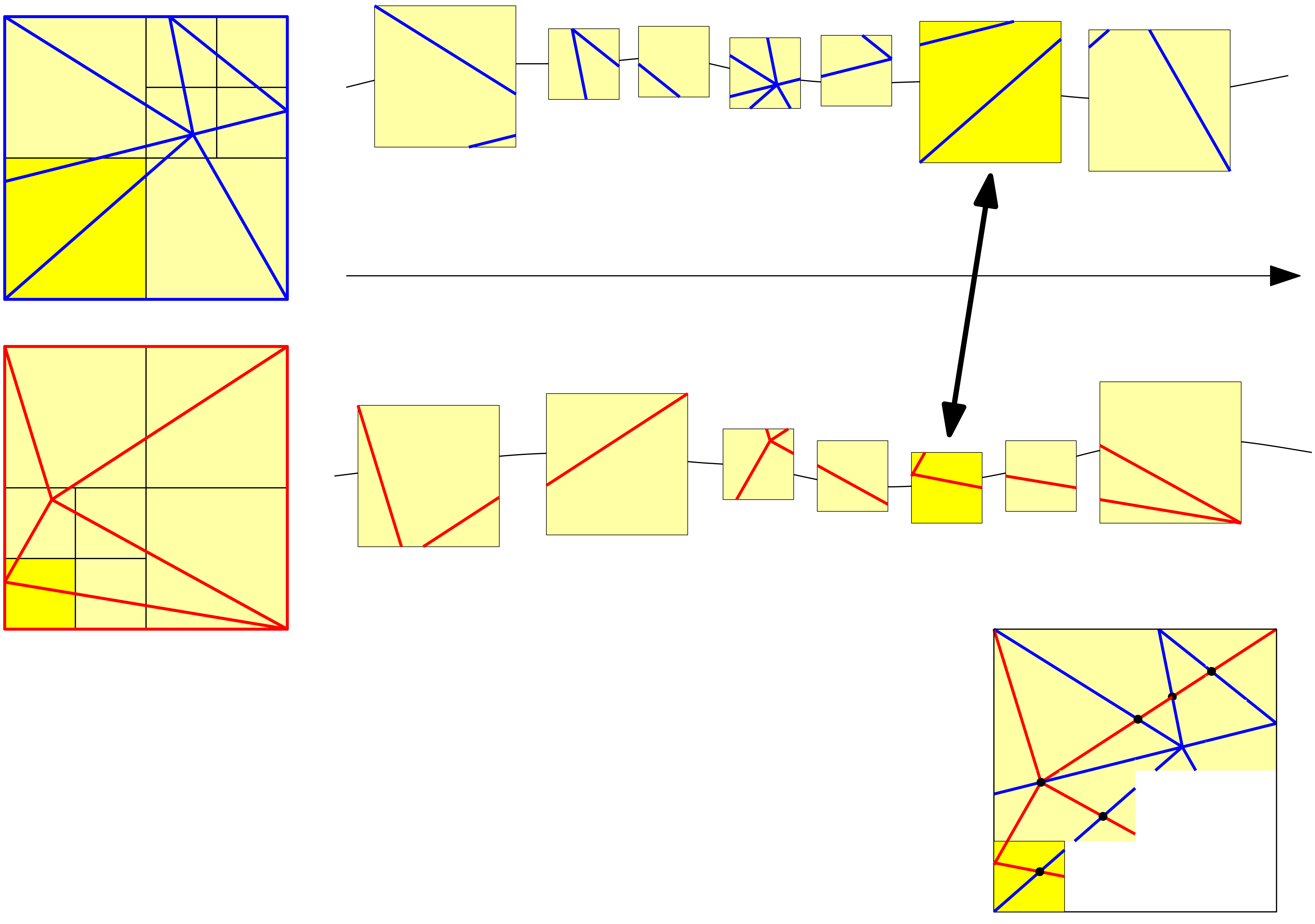

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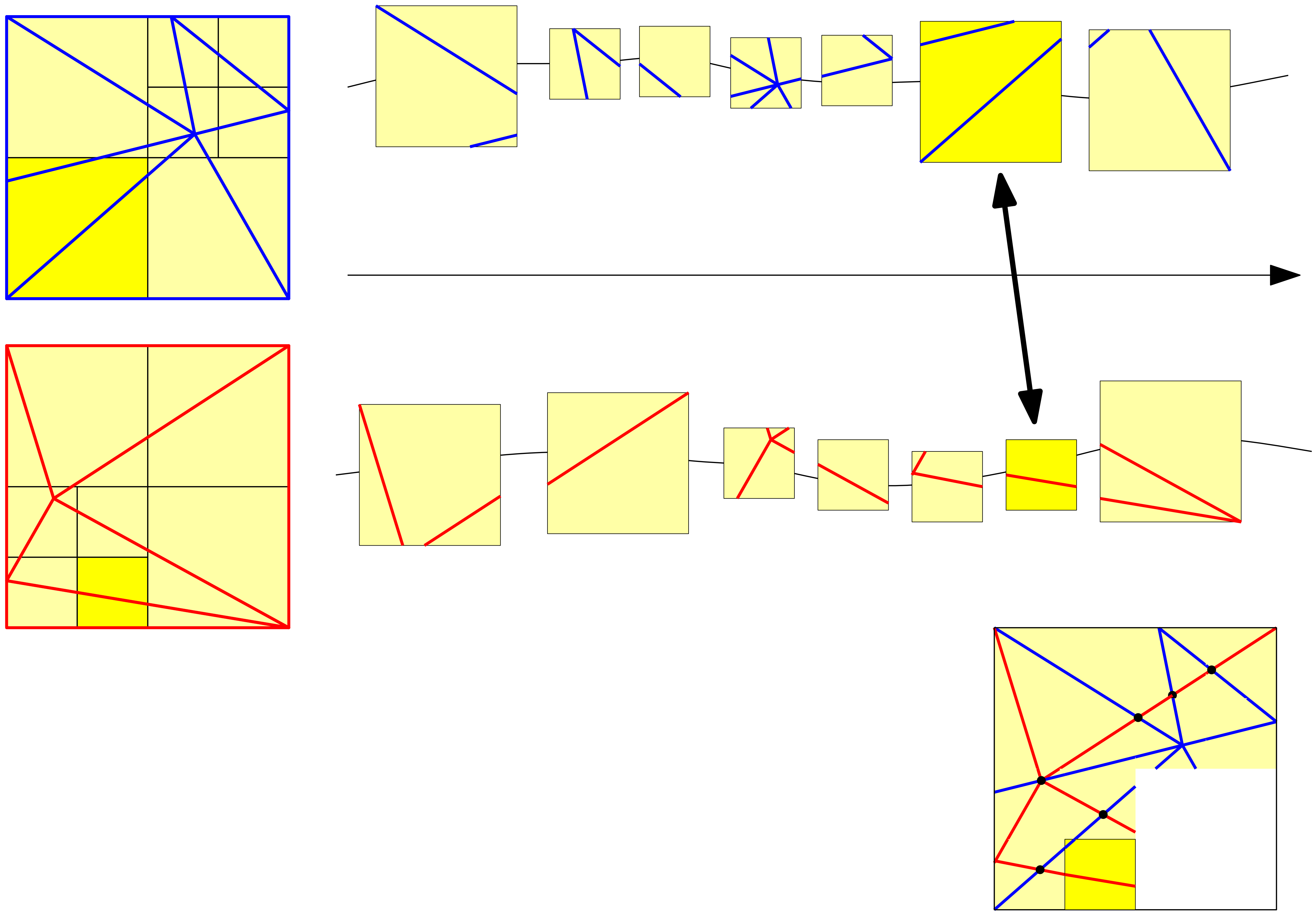

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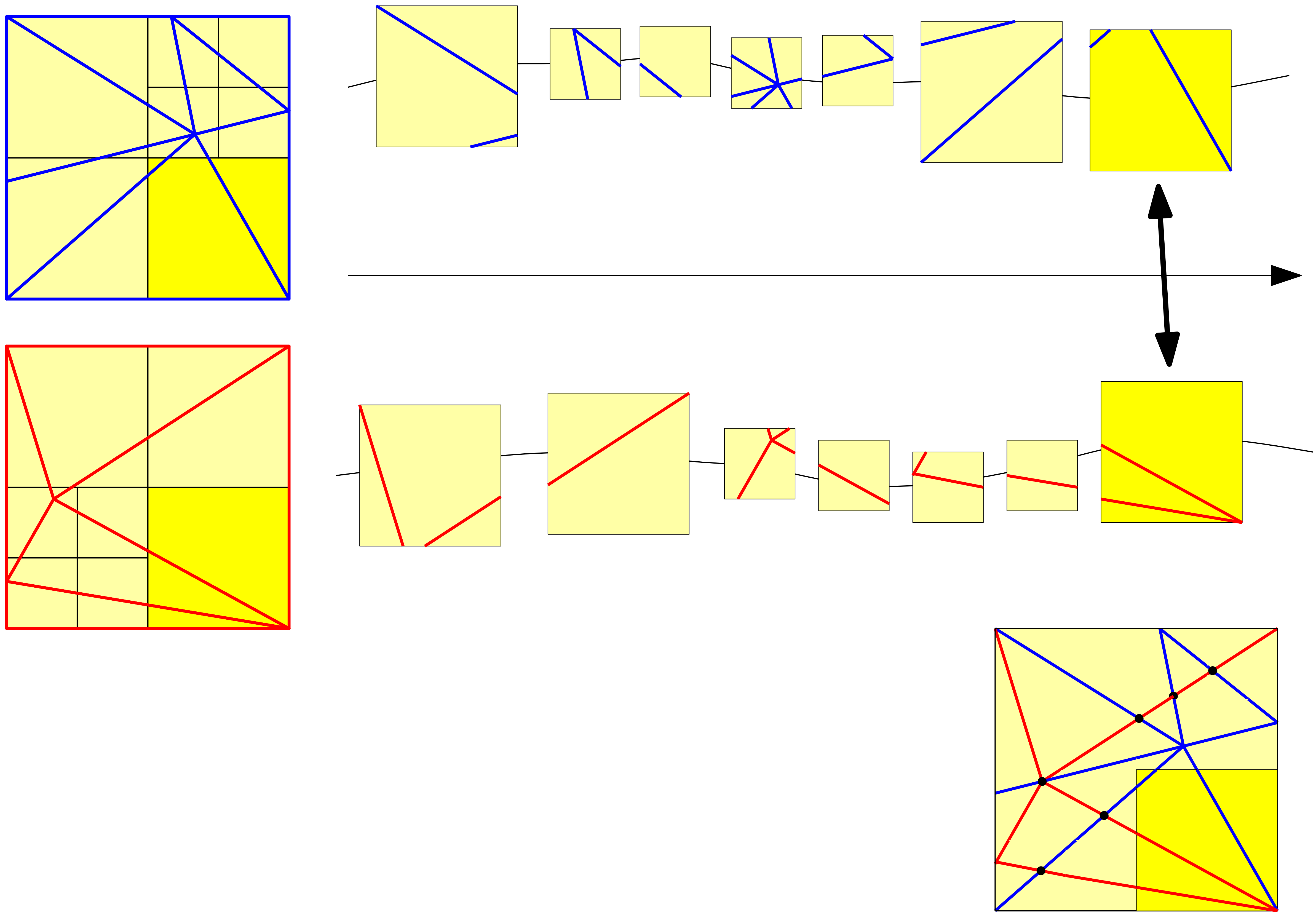

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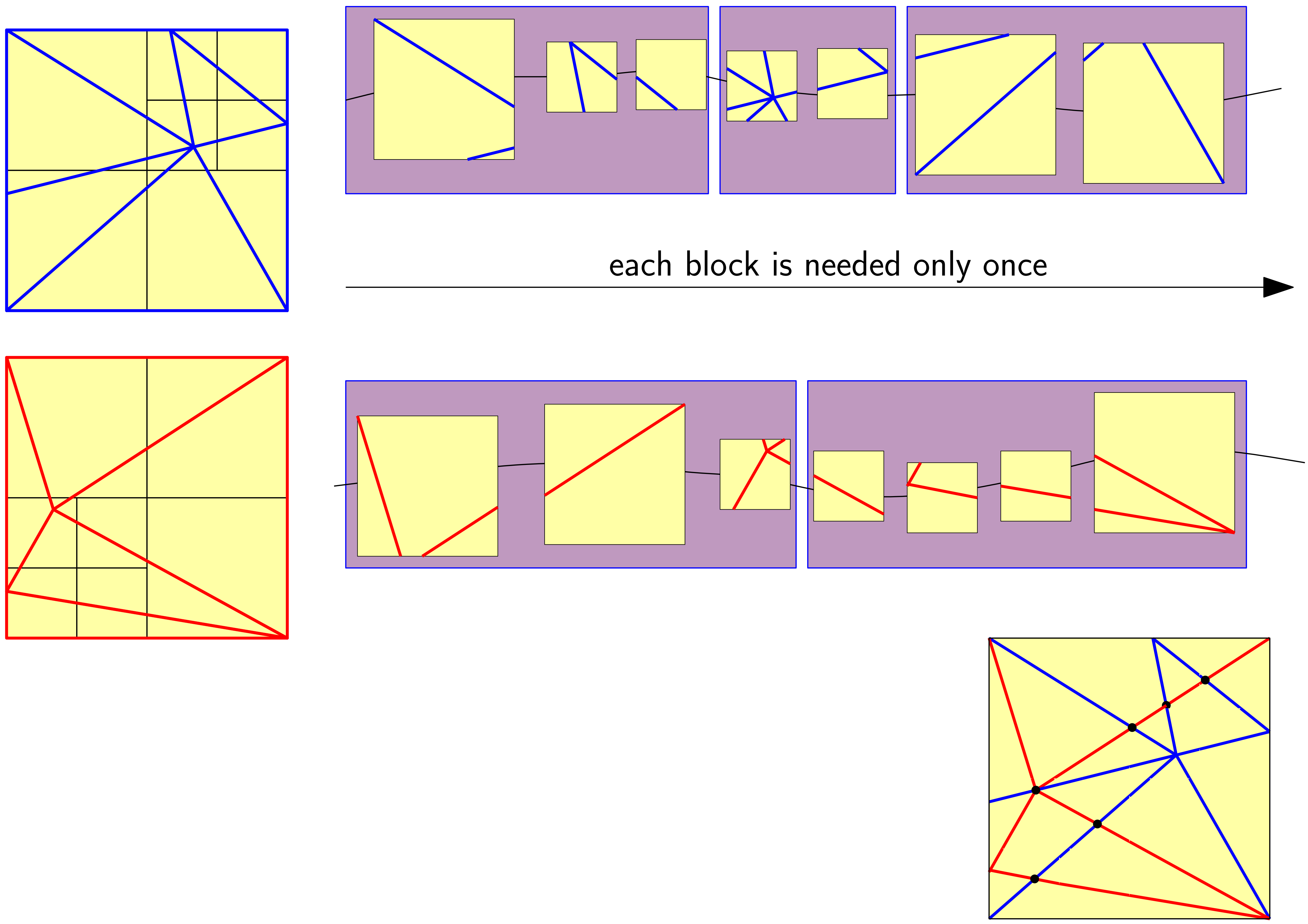

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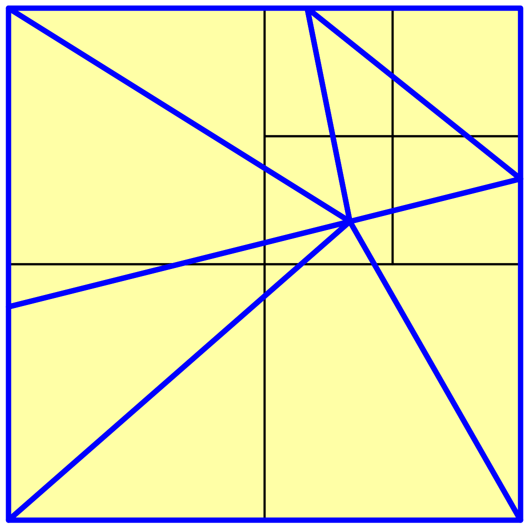
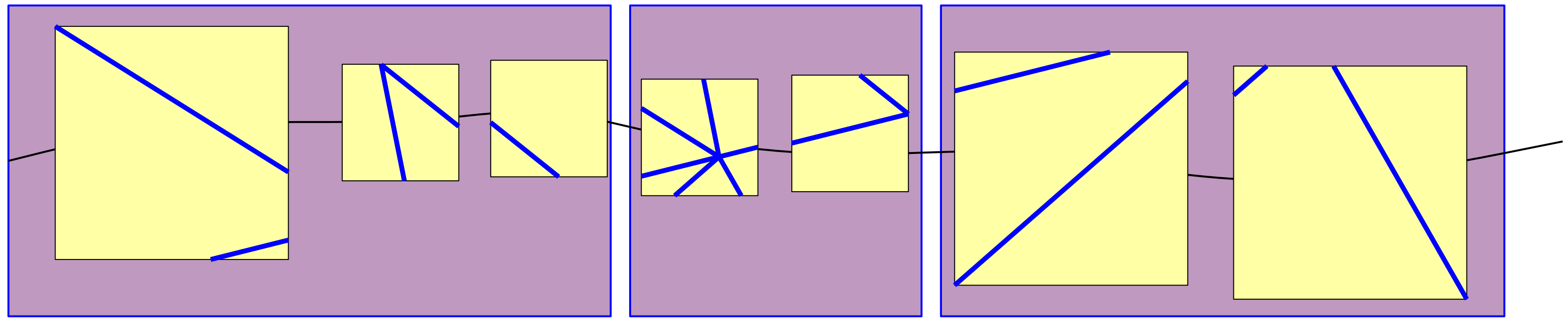
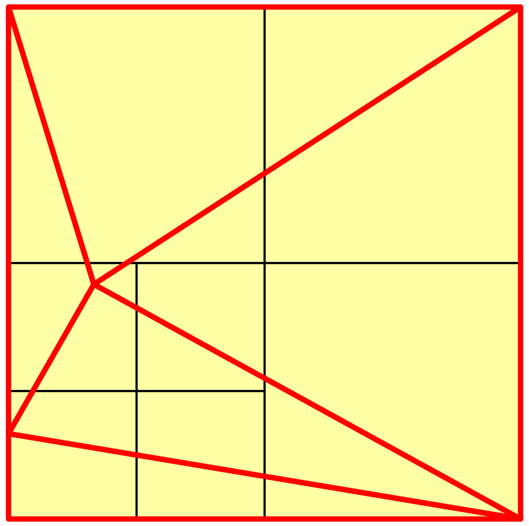
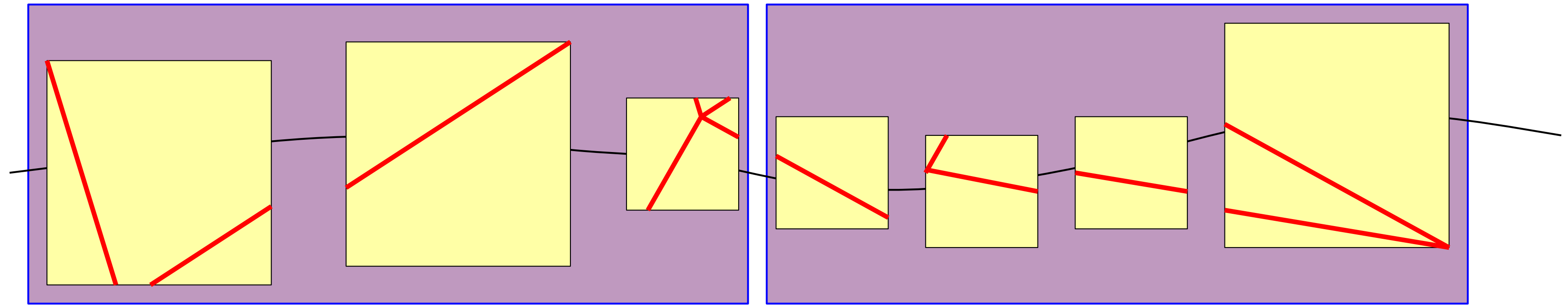




each block is needed only once

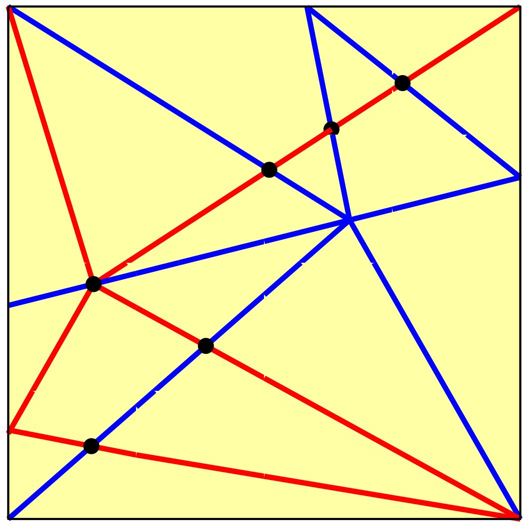




n: number of triangles; B: disk block size

Ideally: $O(n)$ quadtree cells, $O(1)$ edges each

$\rightarrow$ Overlay in $O(scan(n)) = O(n/B)$ I/O's.



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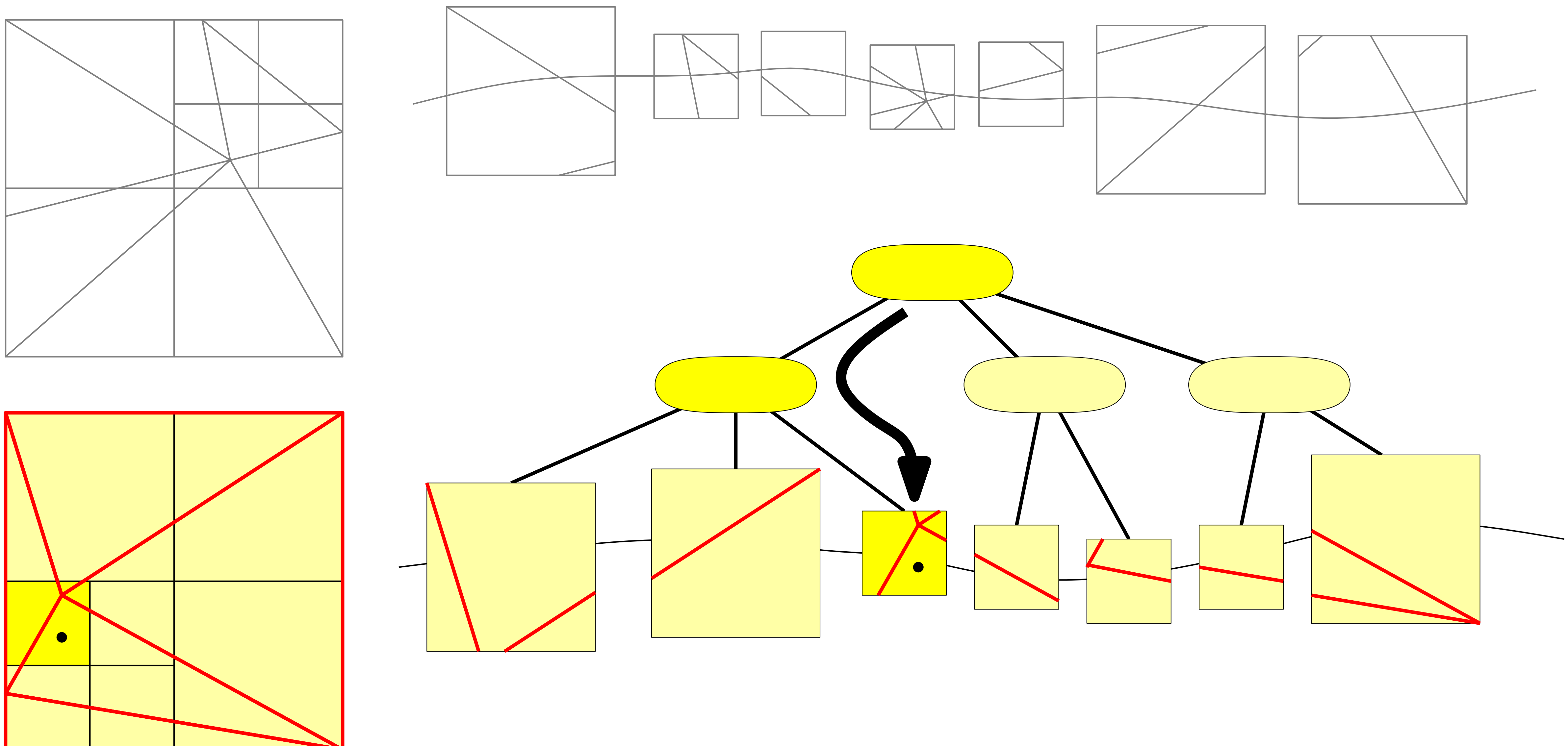

n: number of triangles; B: disk block size

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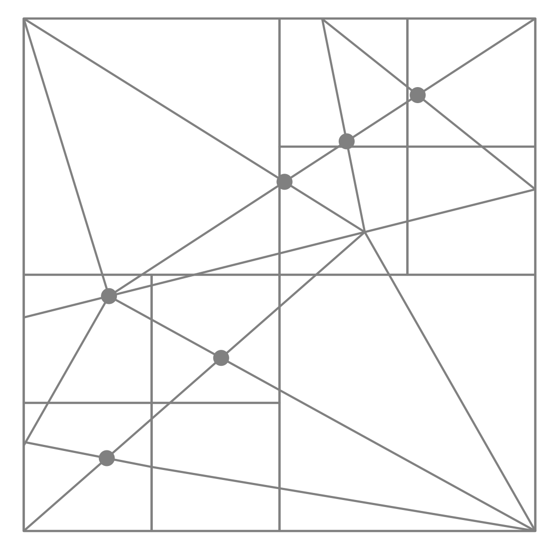
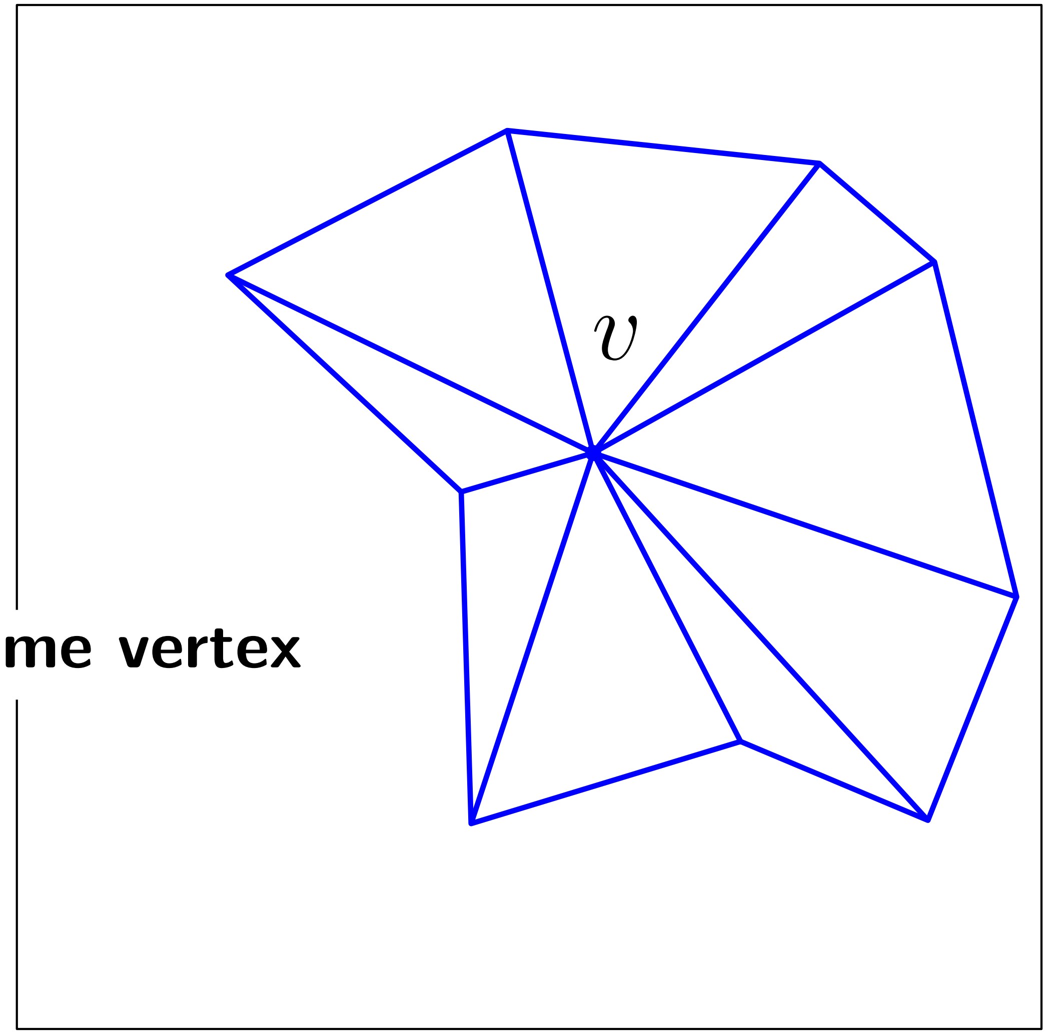
$\longrightarrow$ Overlay in $O(scan(n)) = O(n/B)$ I/O's.

$\longrightarrow$ Point location with B-tree in $O(\log_B n)$ I/O's.



How to get that quadtree in Z-order (for triangulations of unit square)

Input: file with for each vertex its adjacency list.



Algorithm:

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     Stop splitting when all edges incident to same vertex

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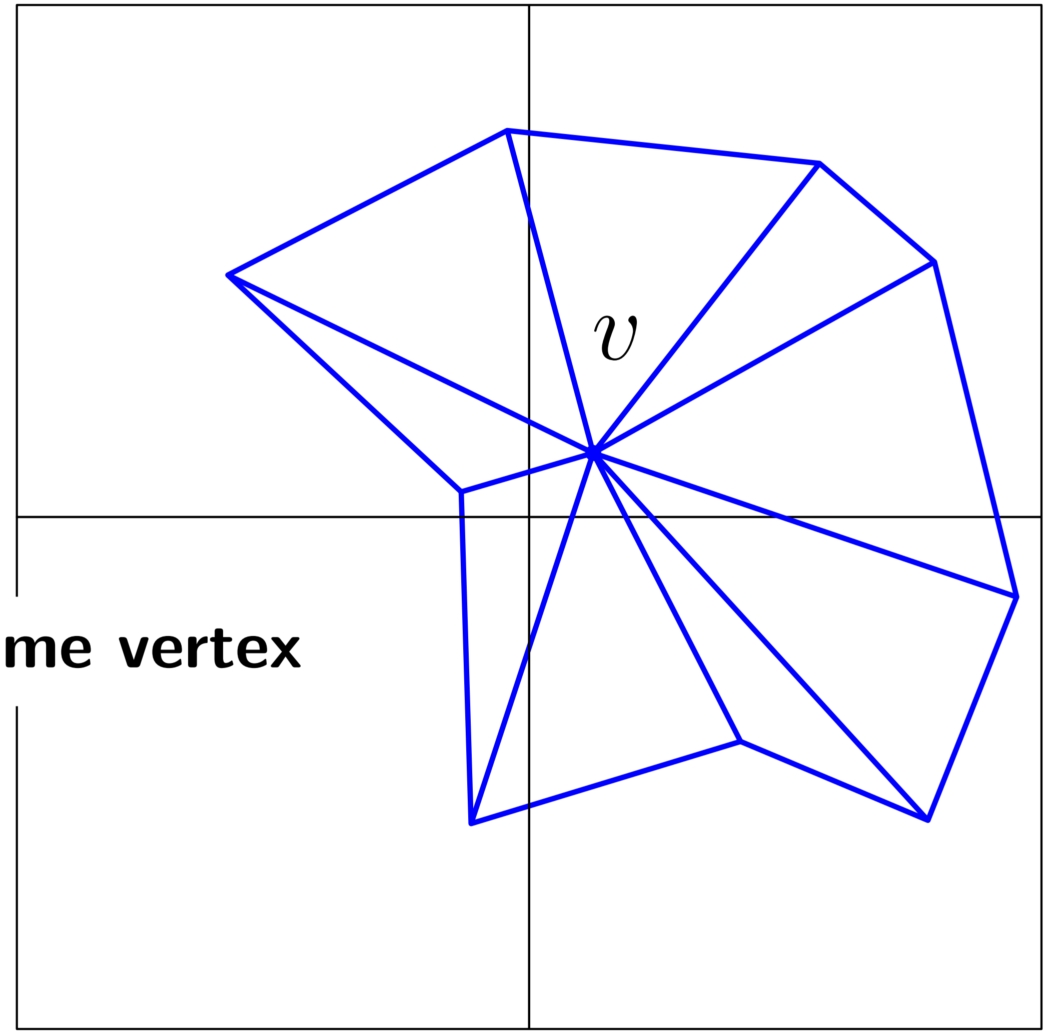

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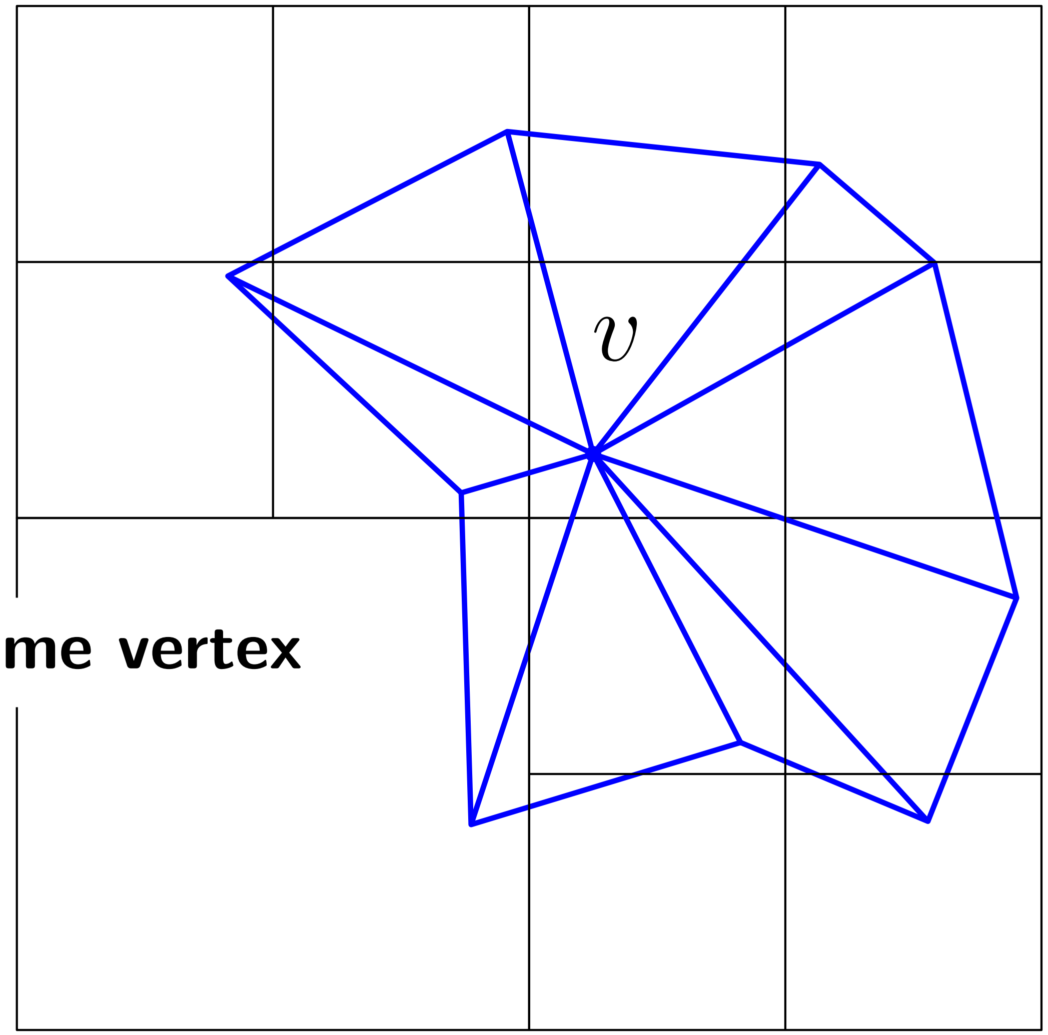

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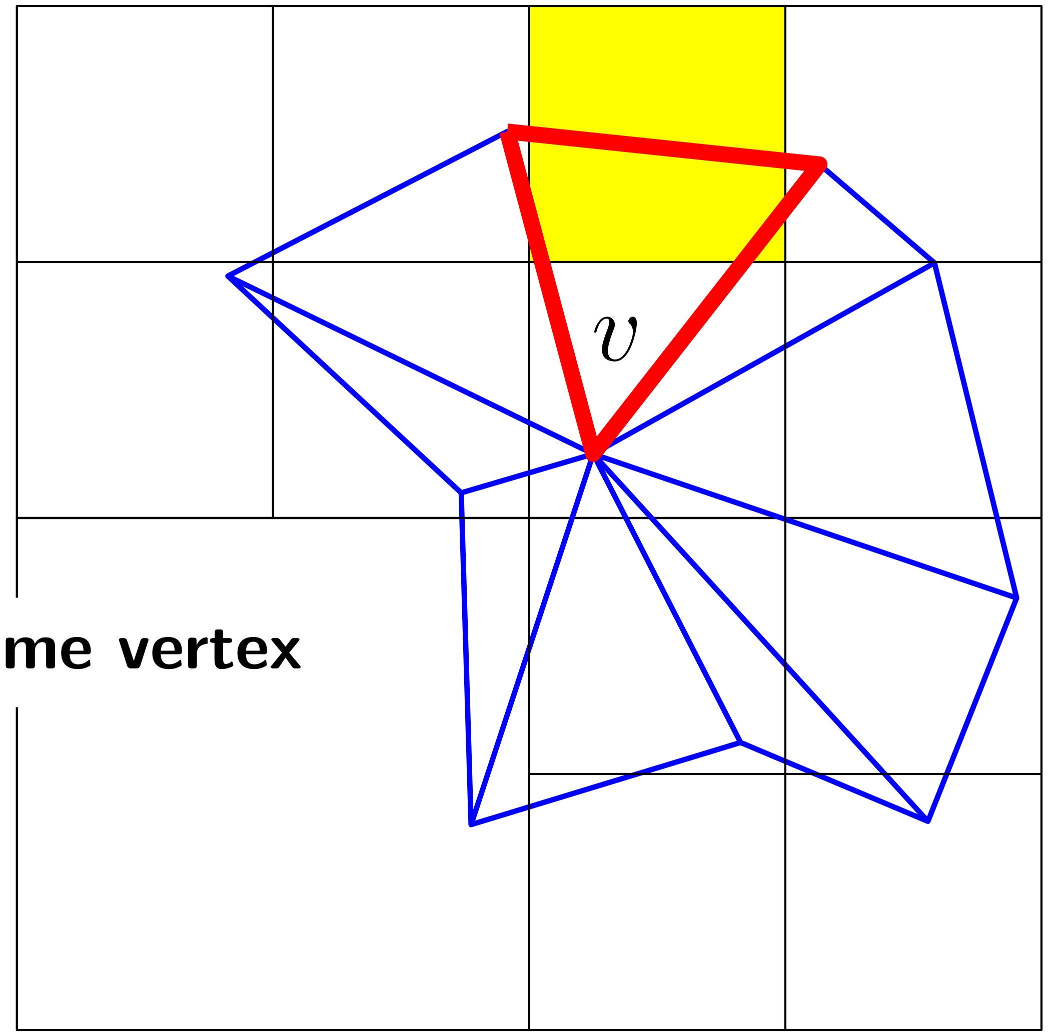

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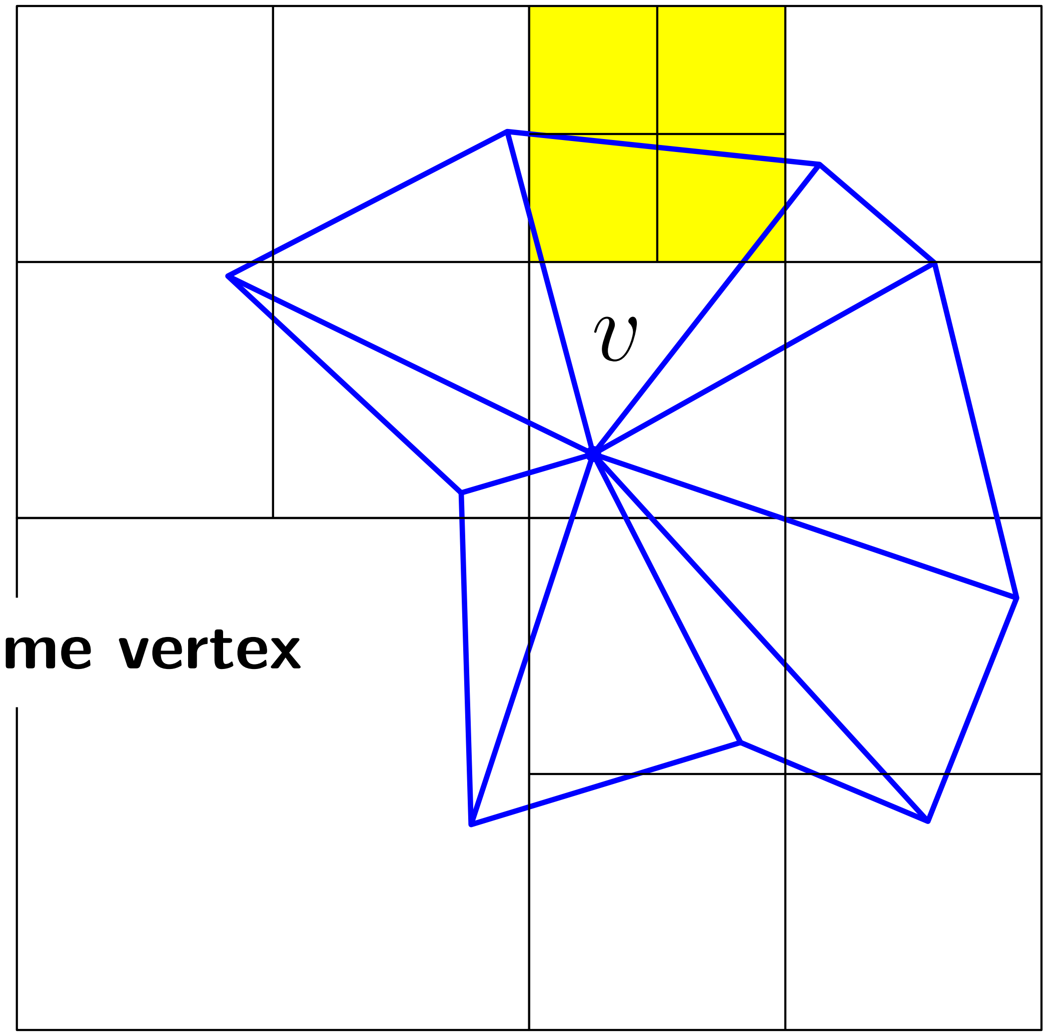

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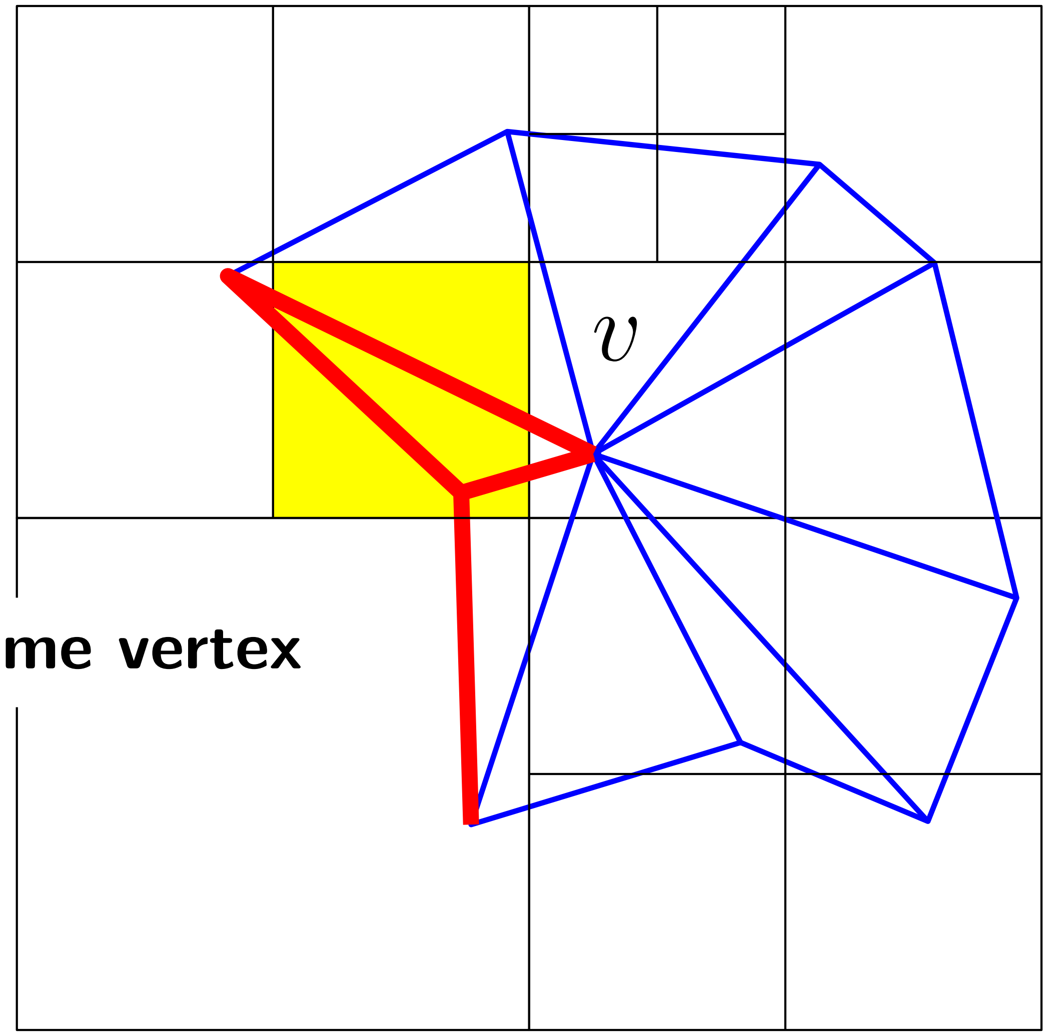

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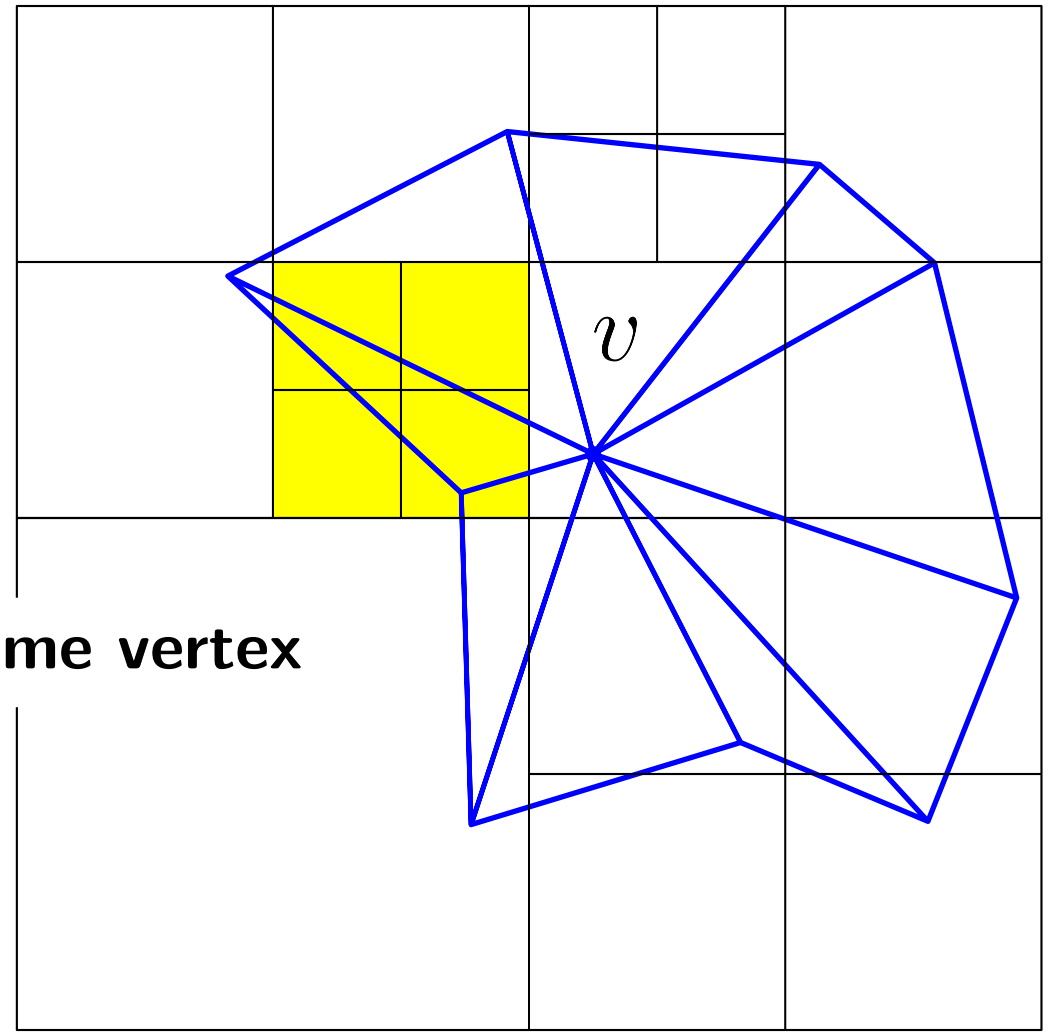

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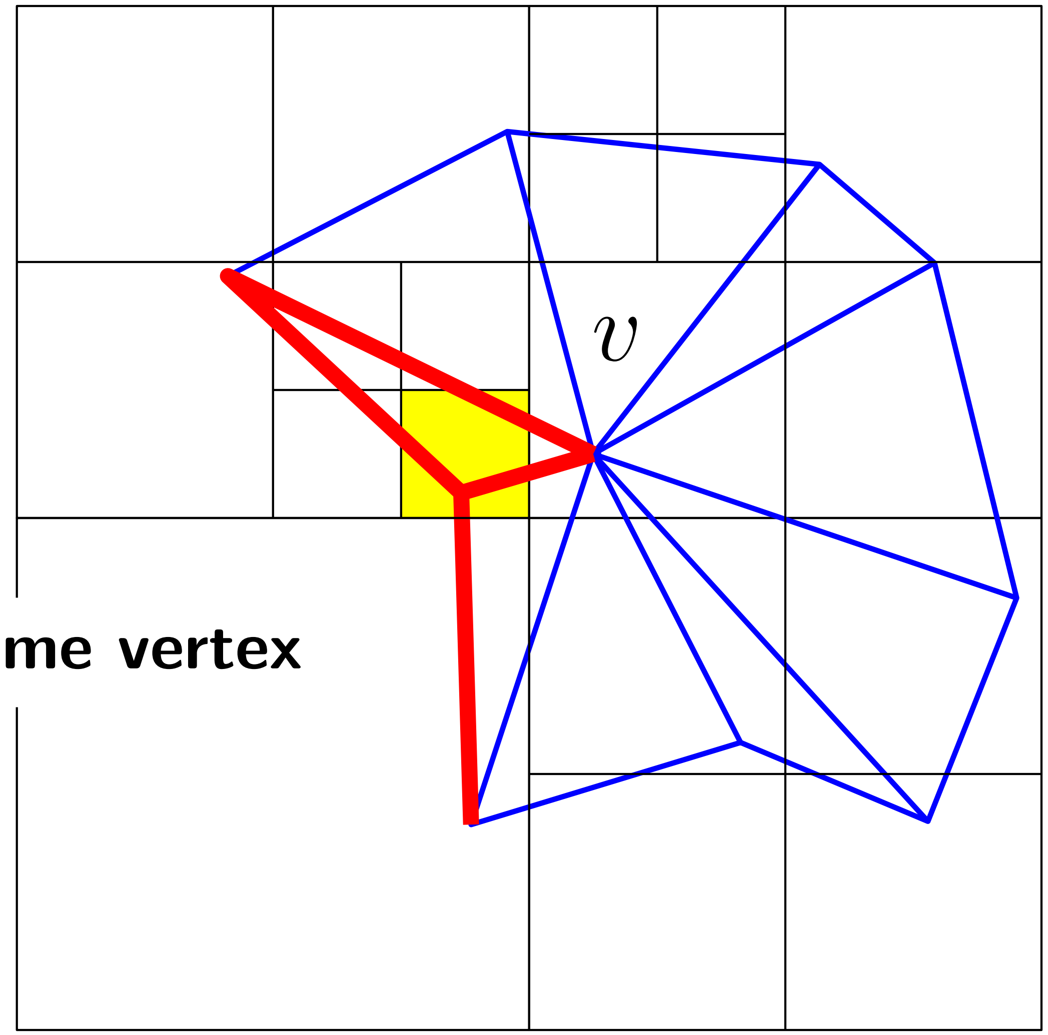

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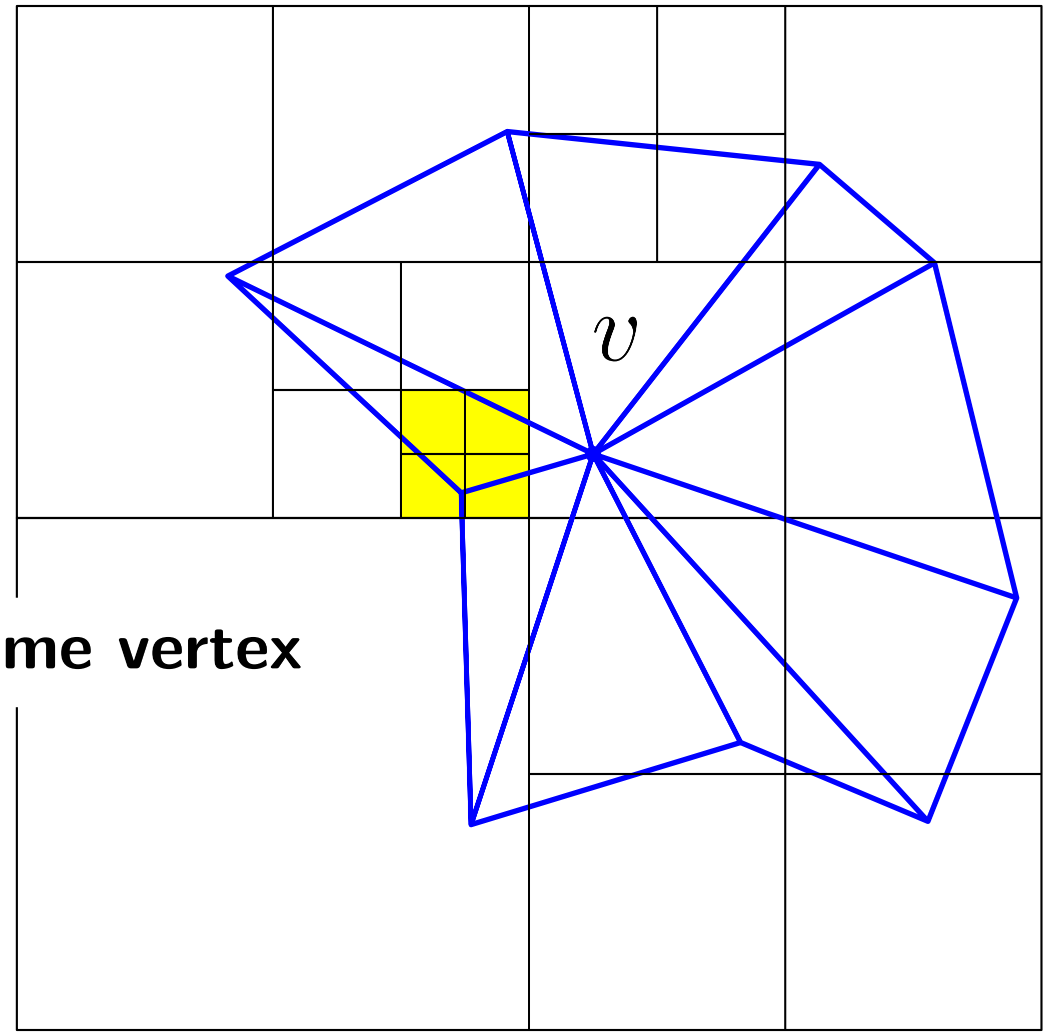

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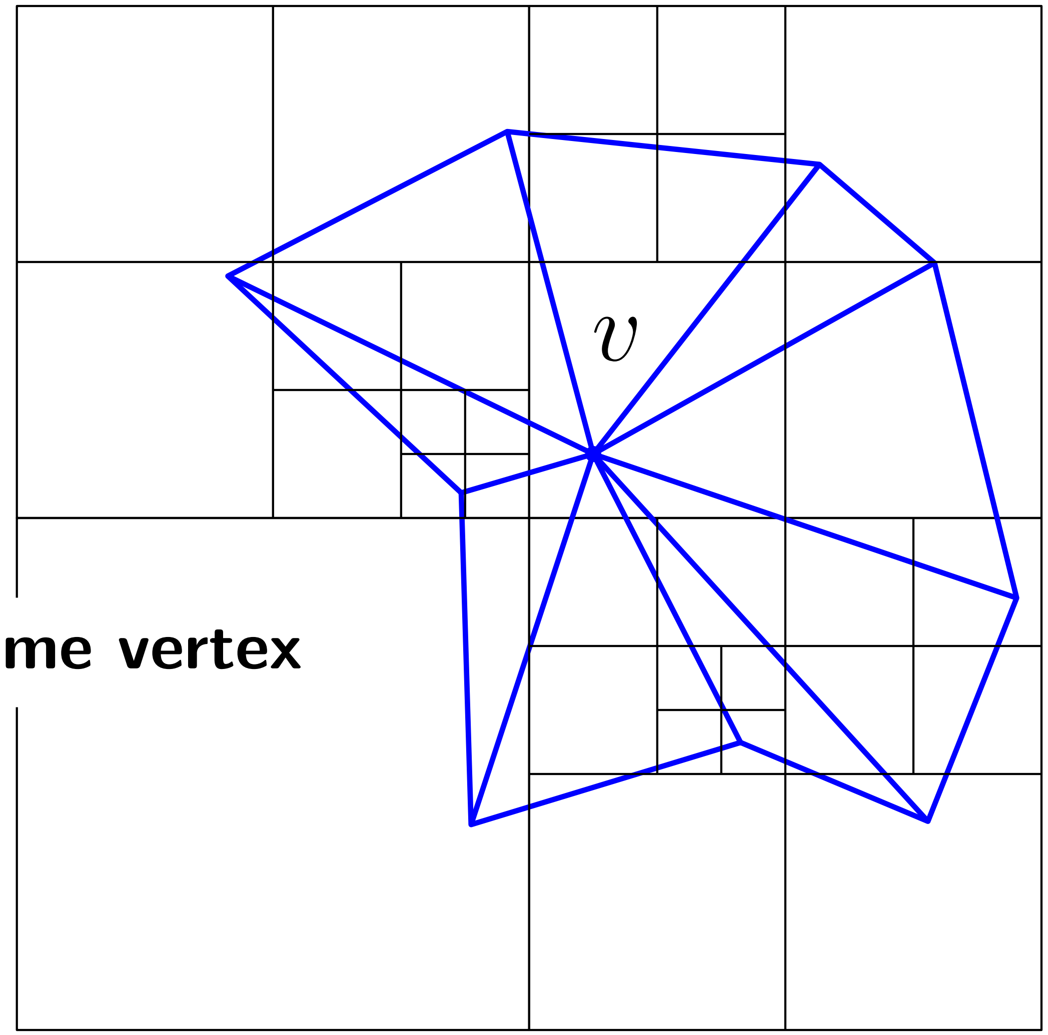

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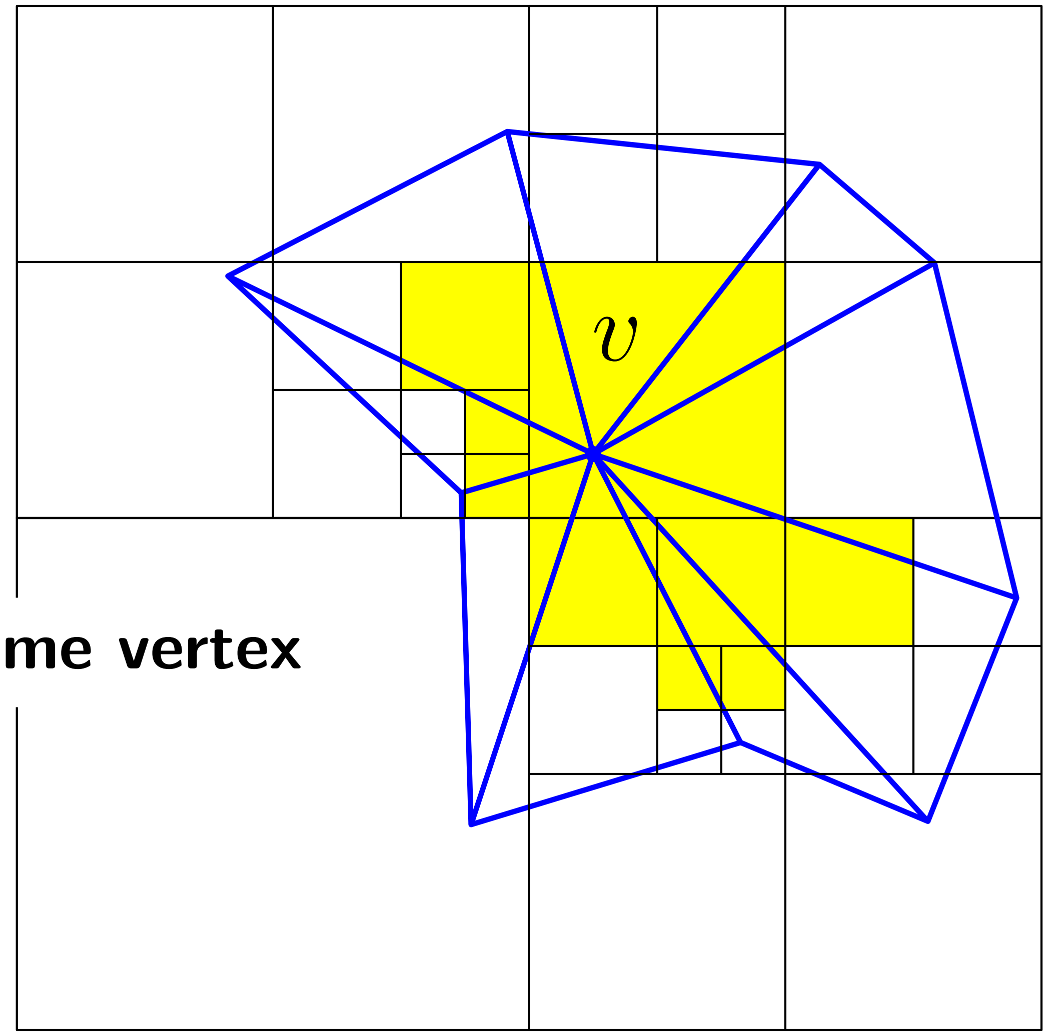

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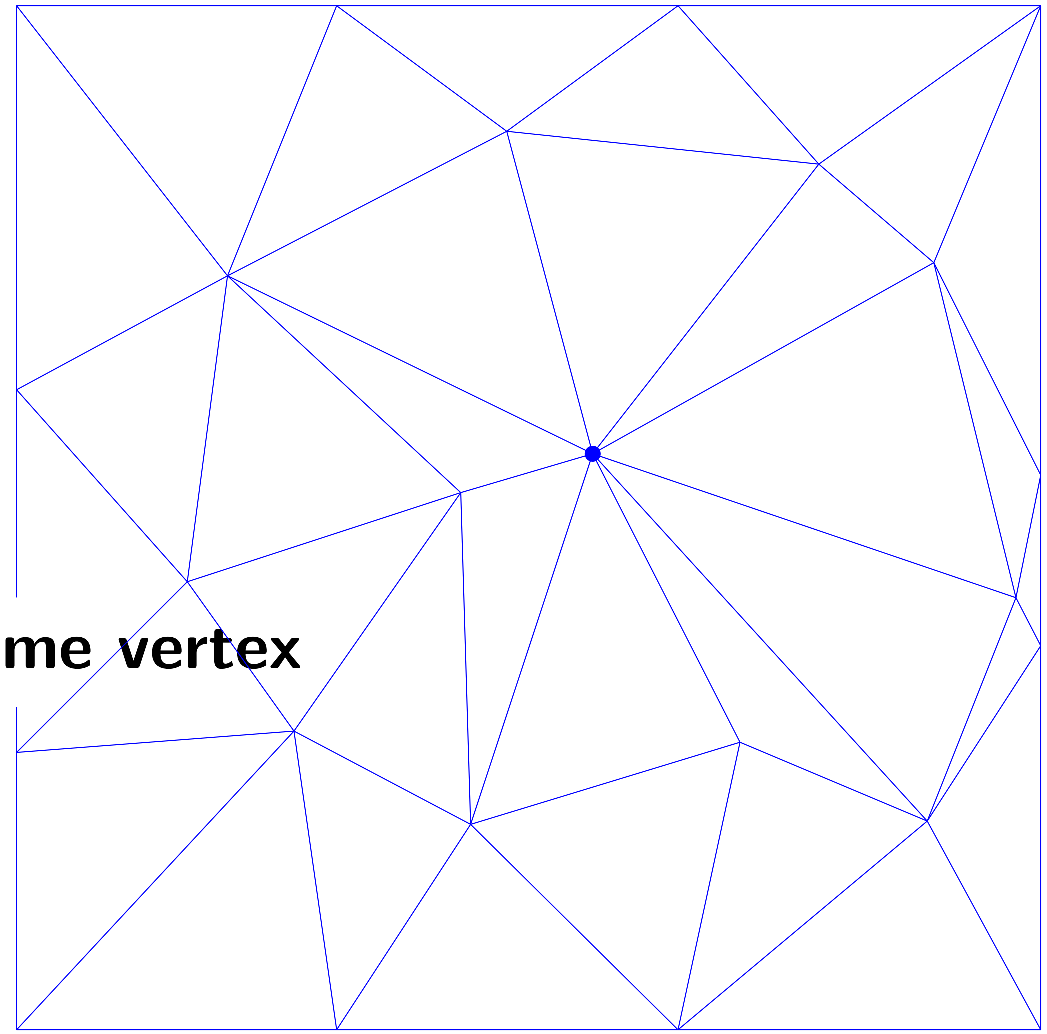

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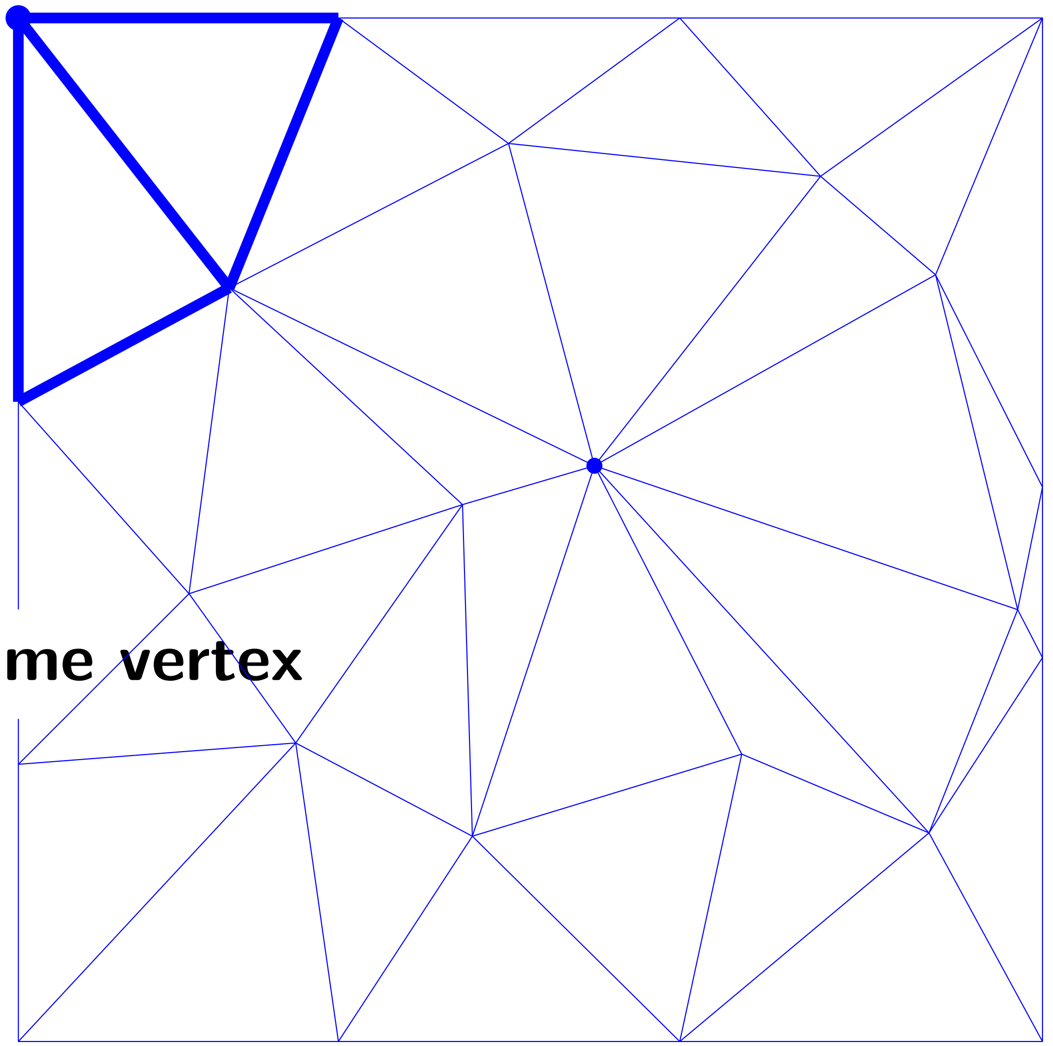

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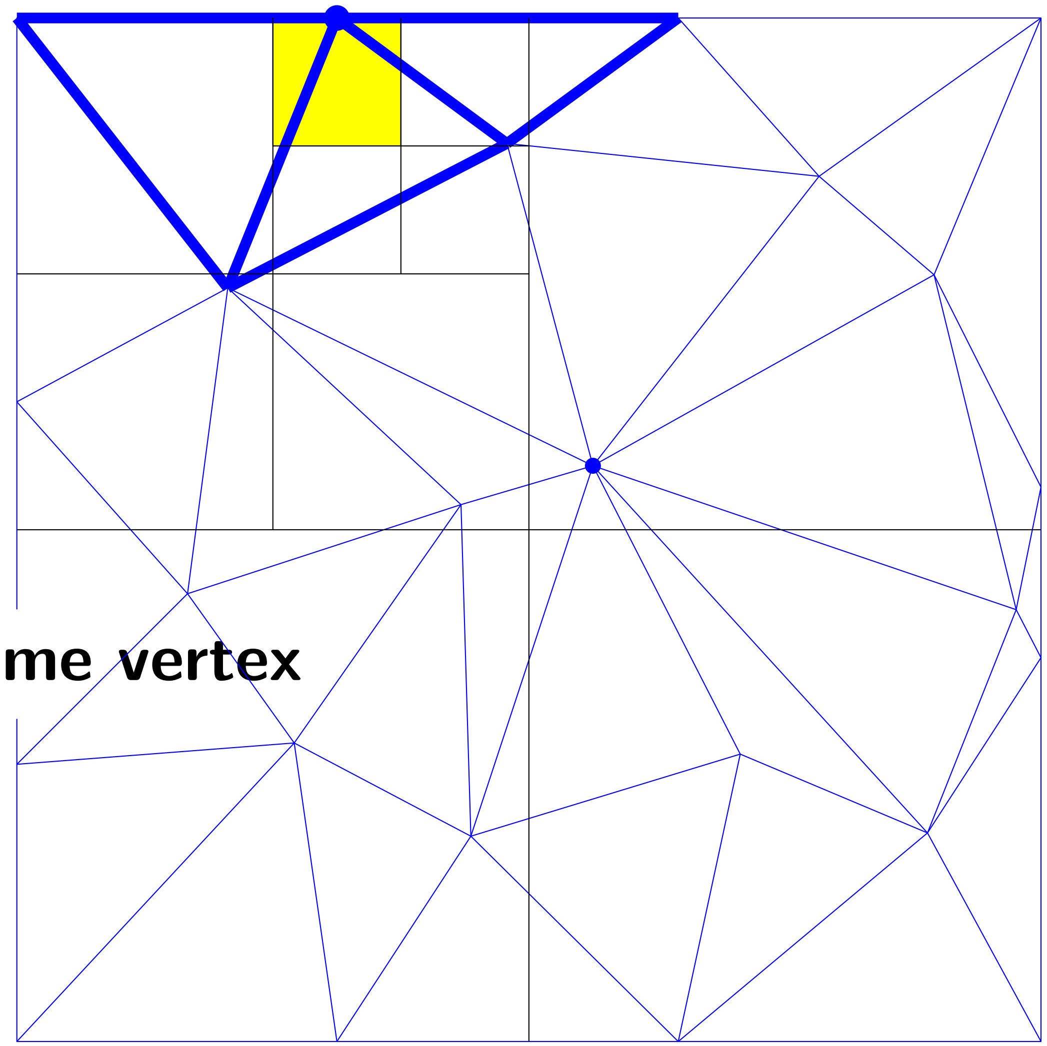

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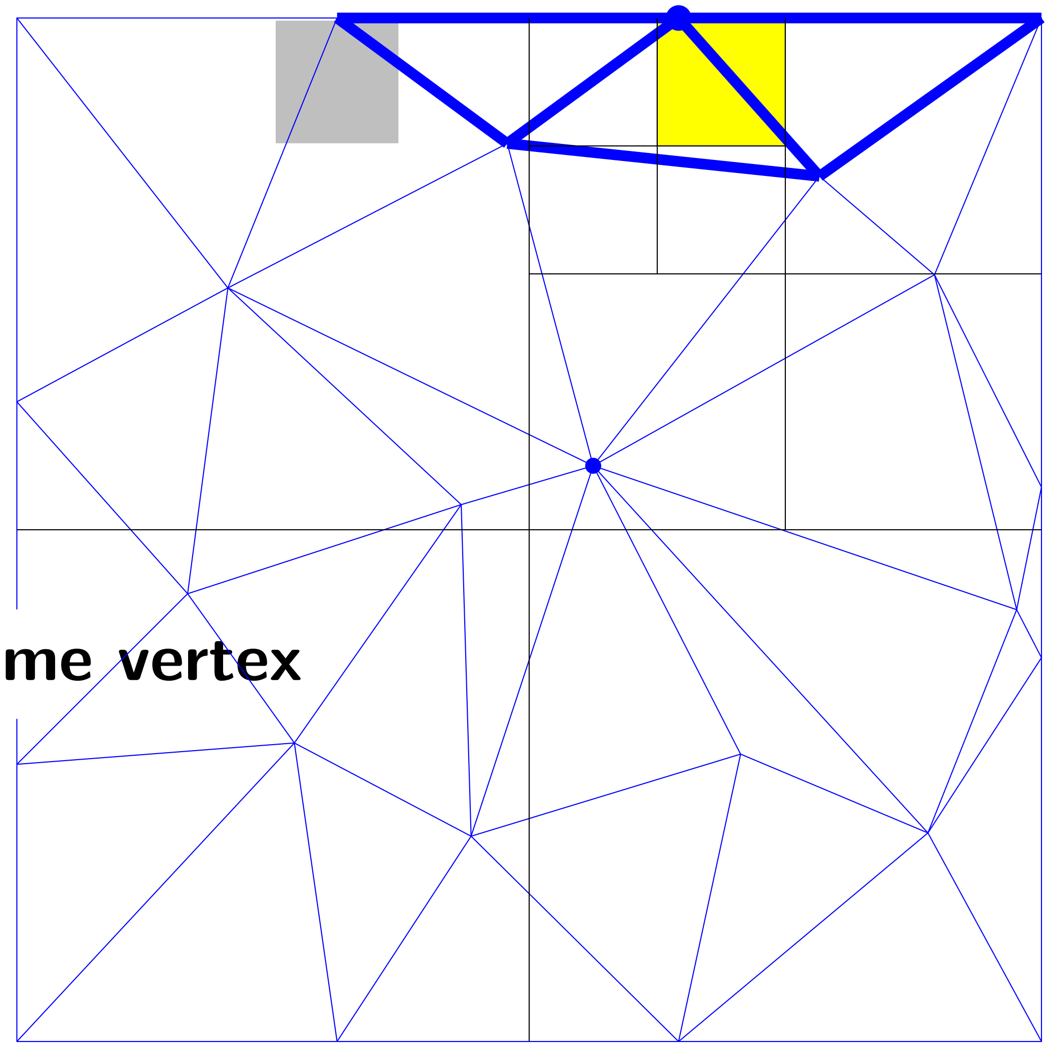

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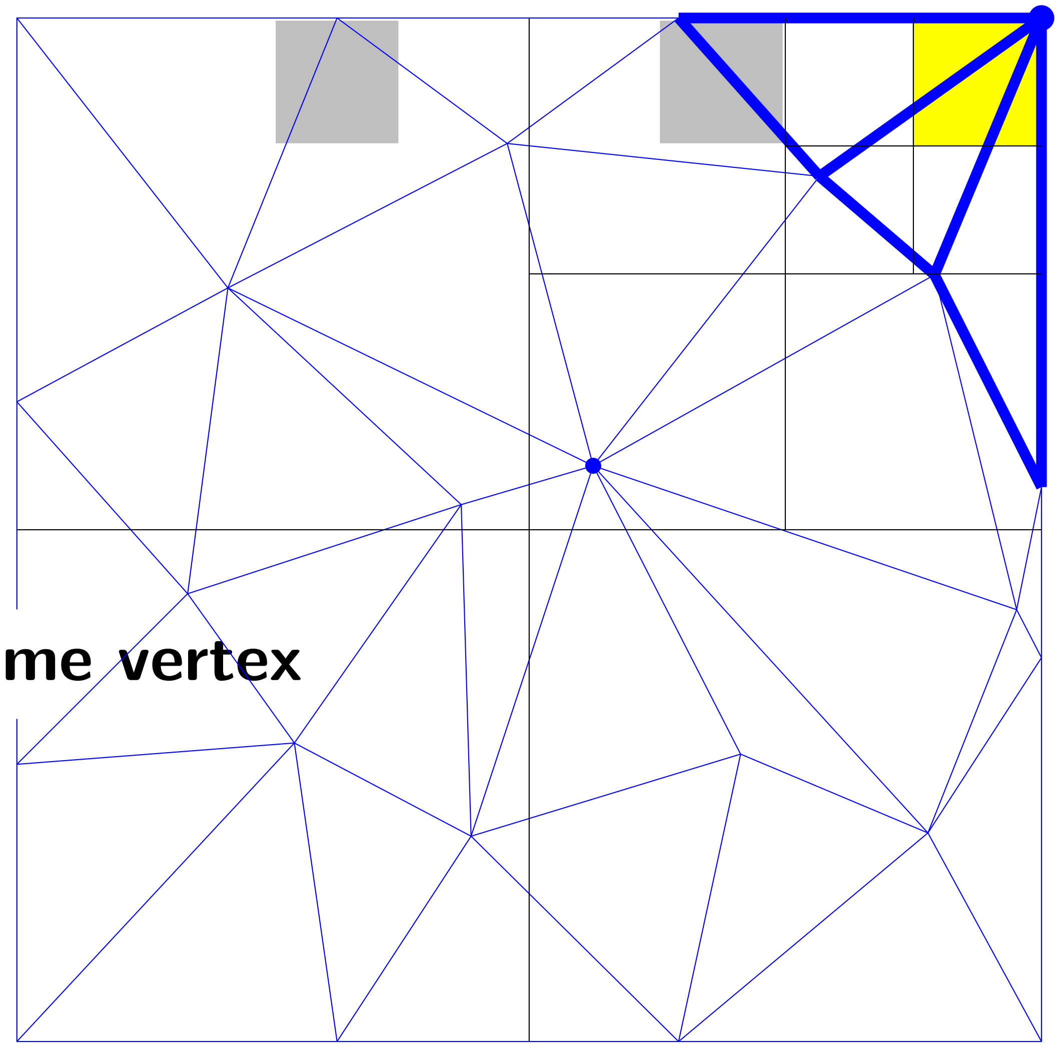

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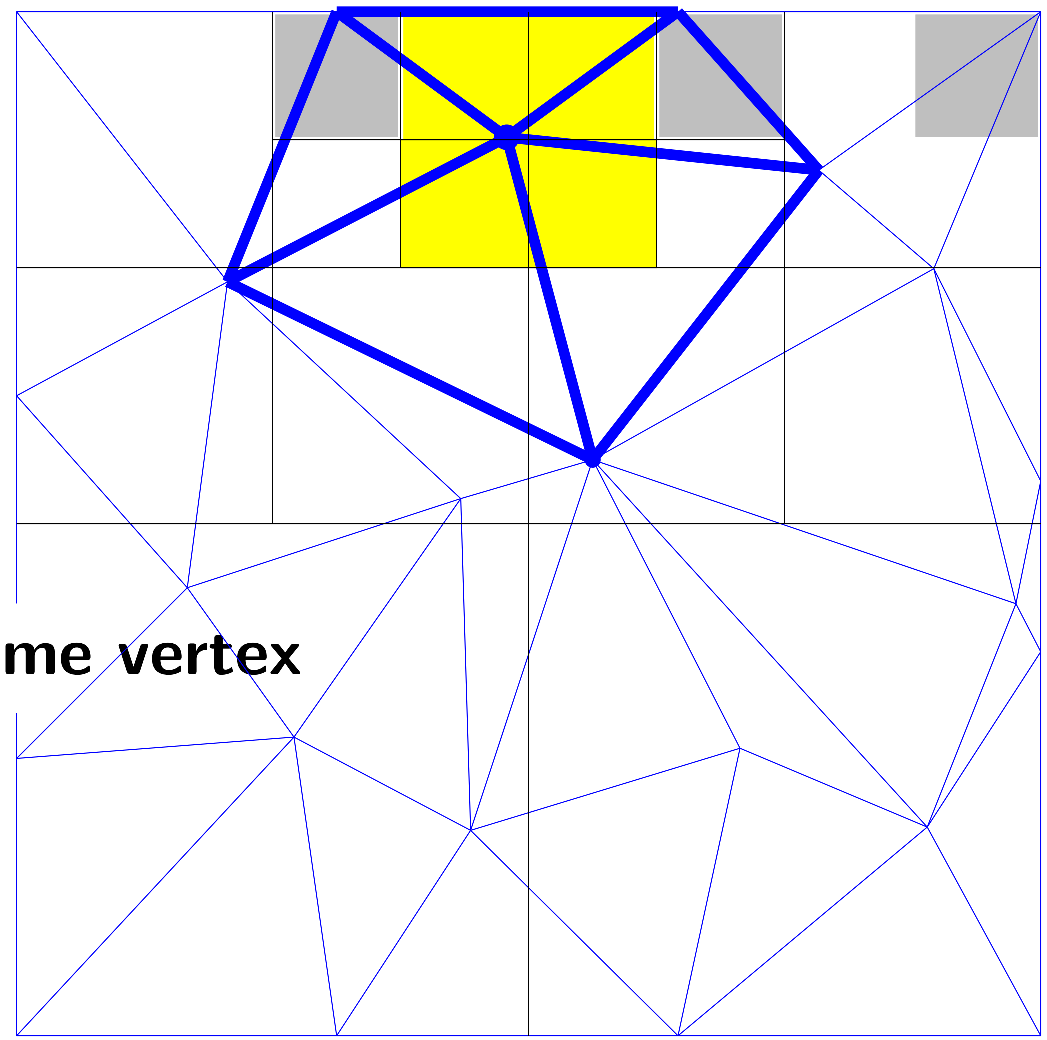

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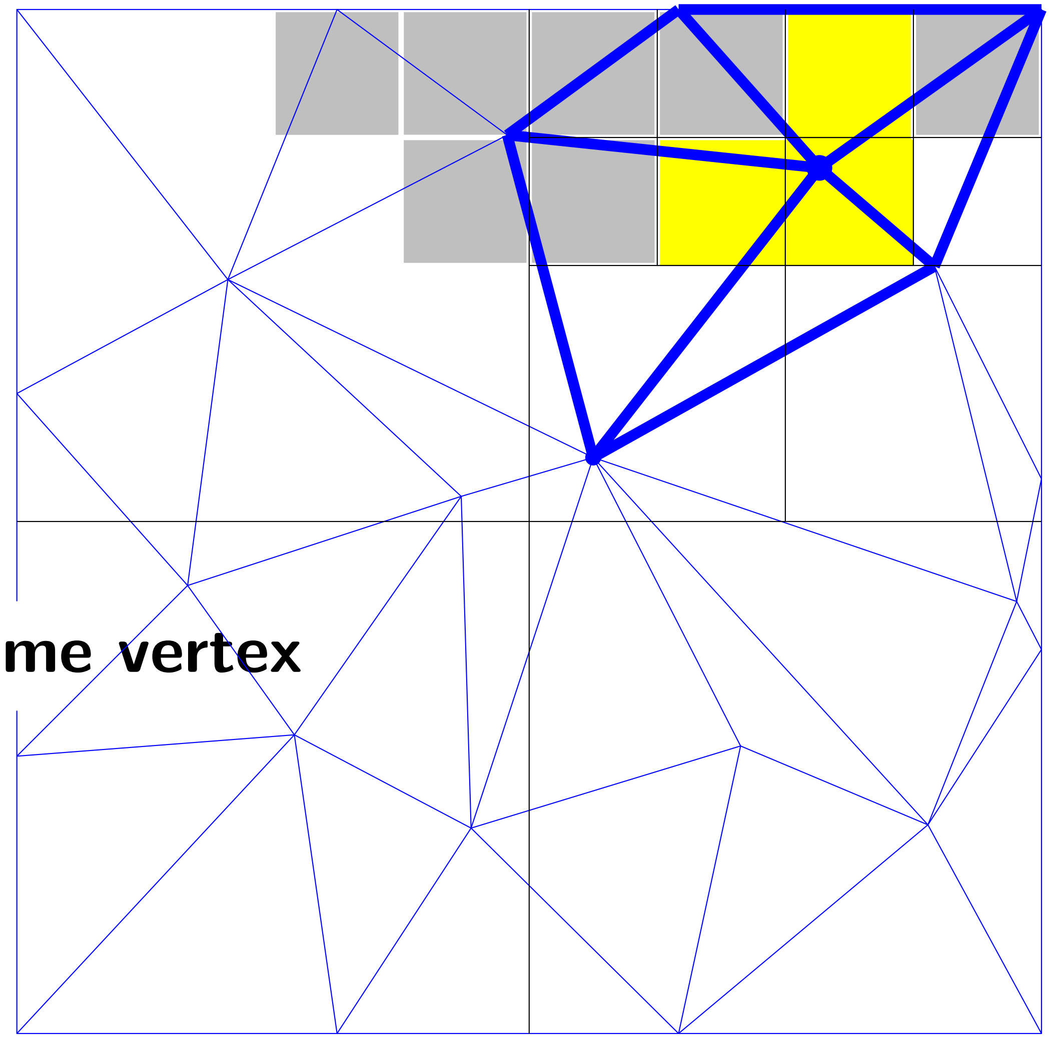

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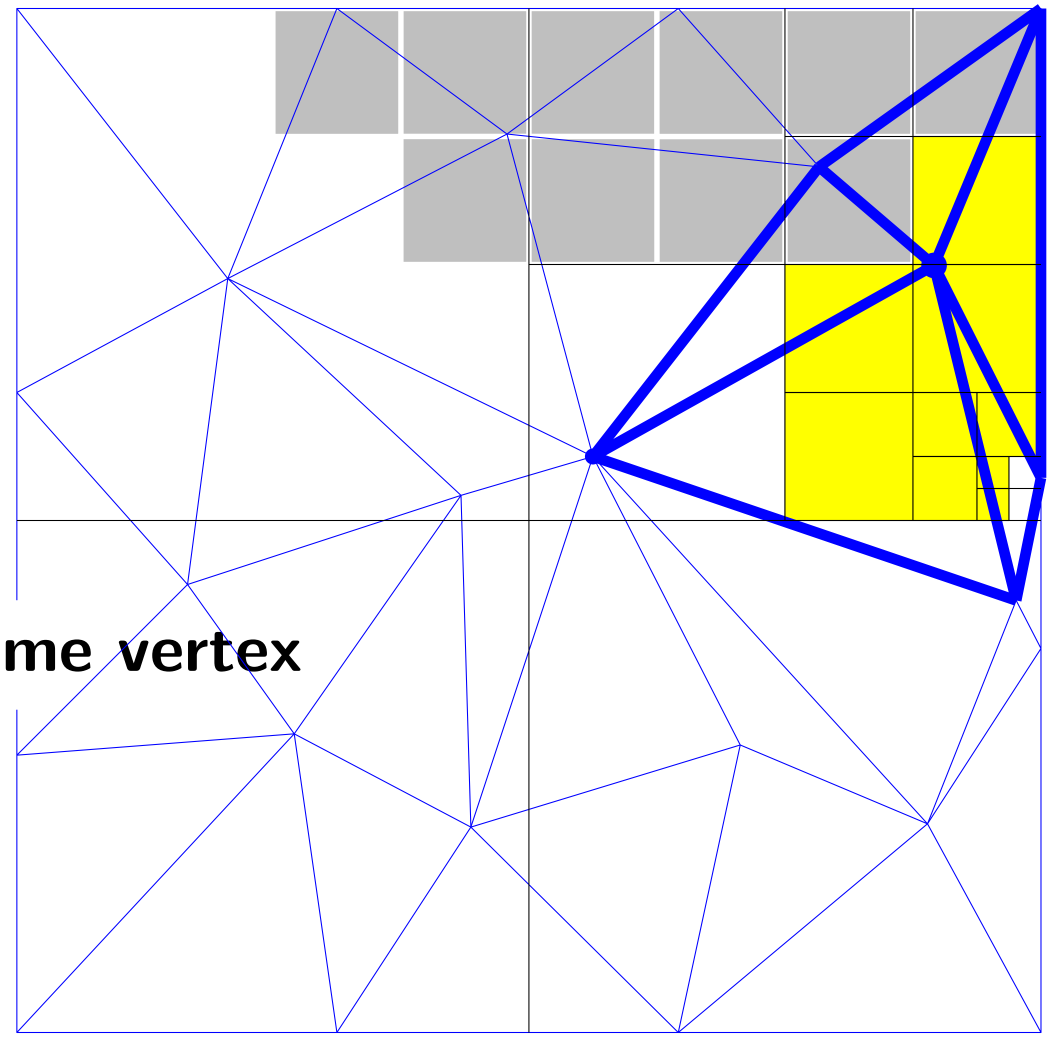

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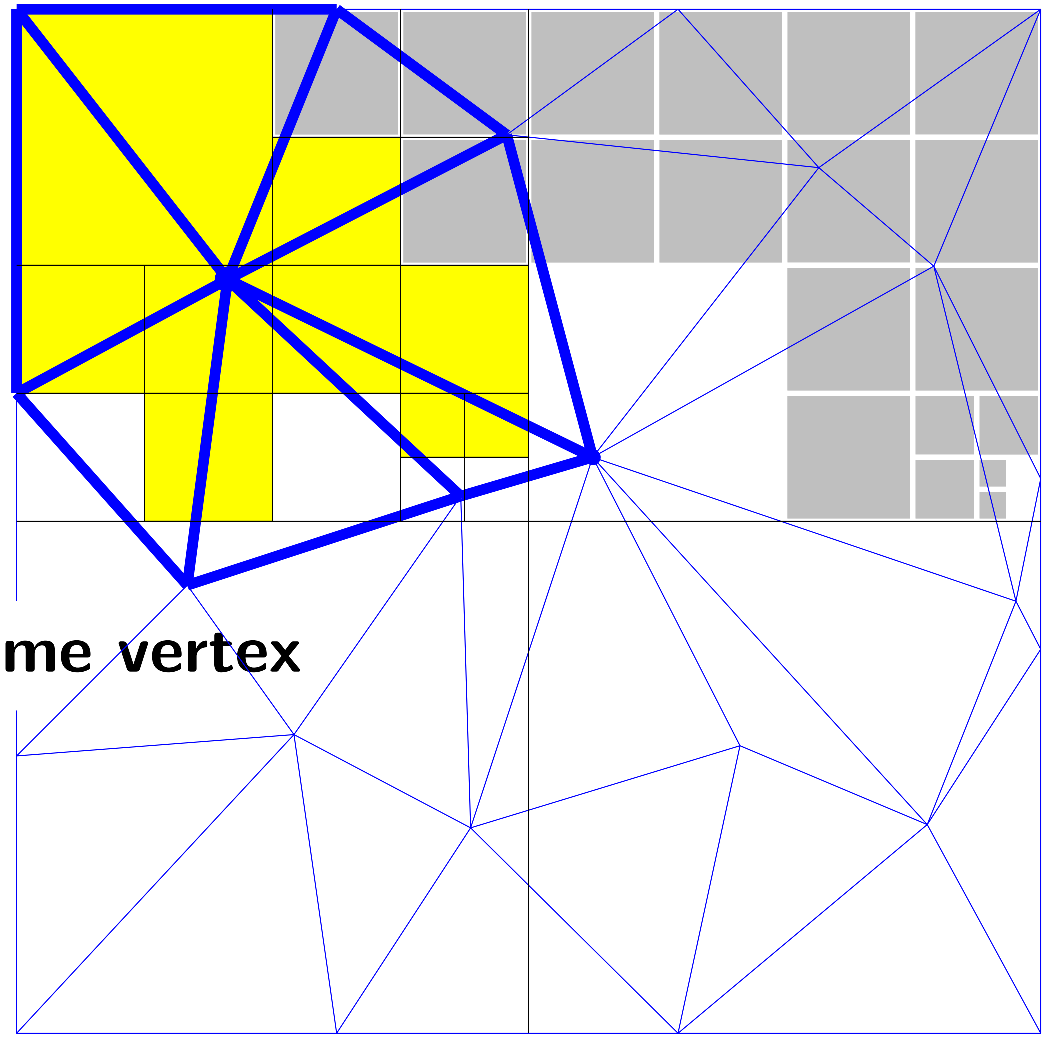

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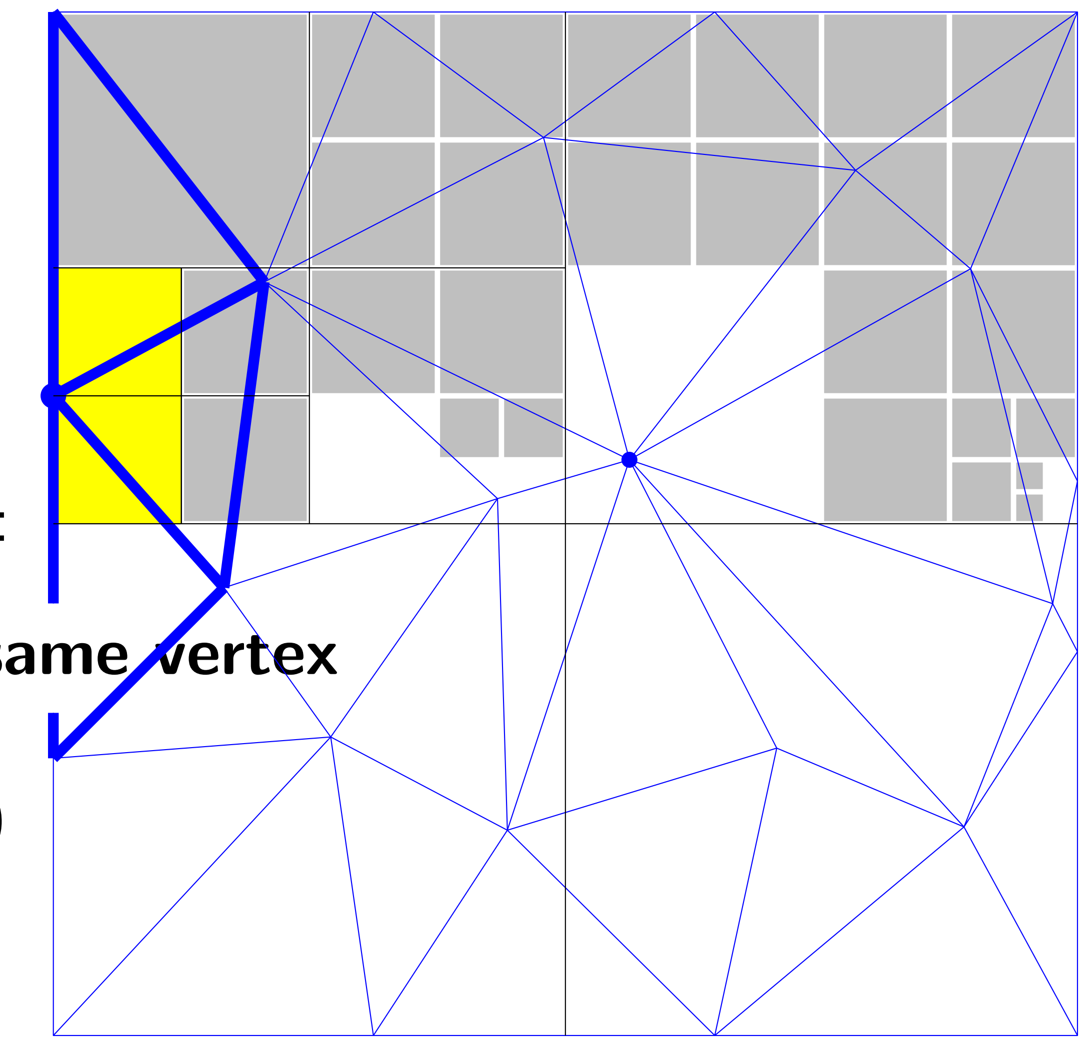

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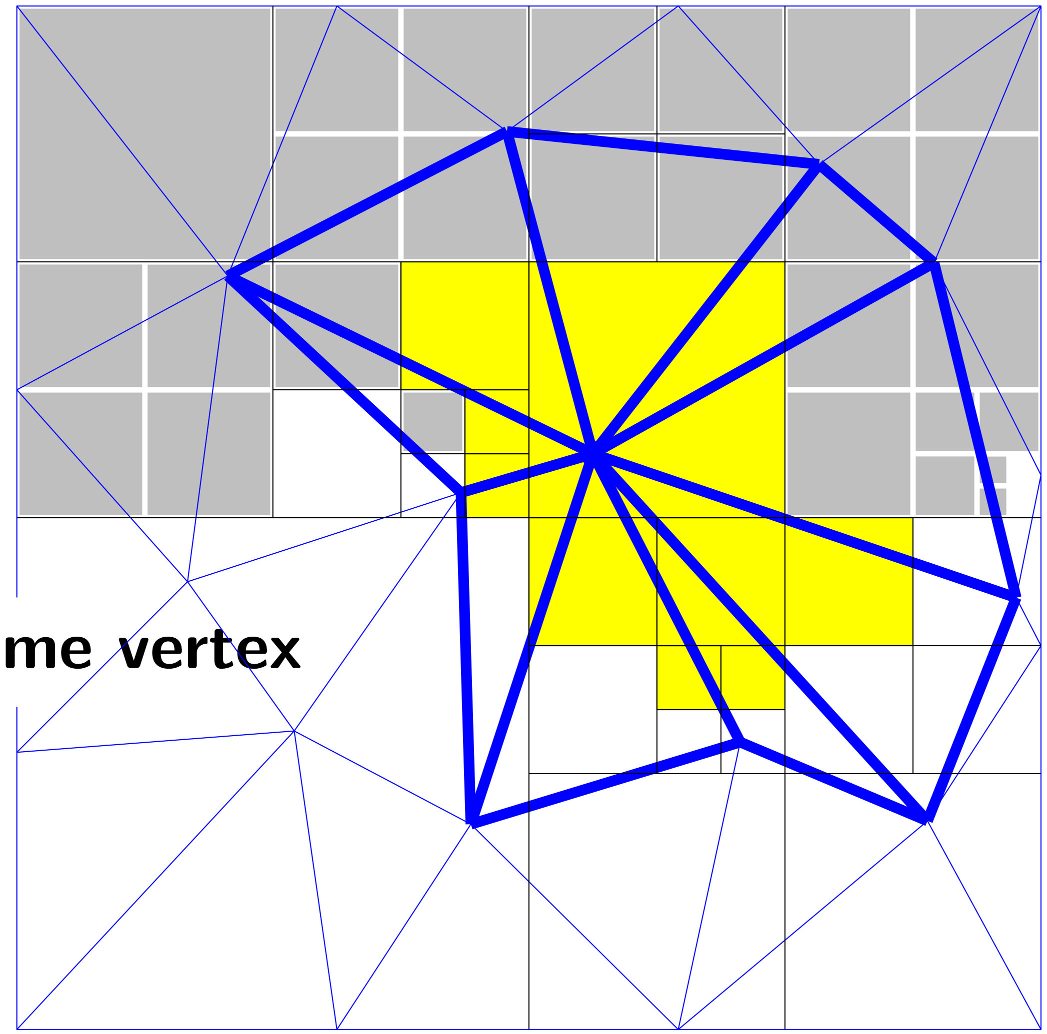

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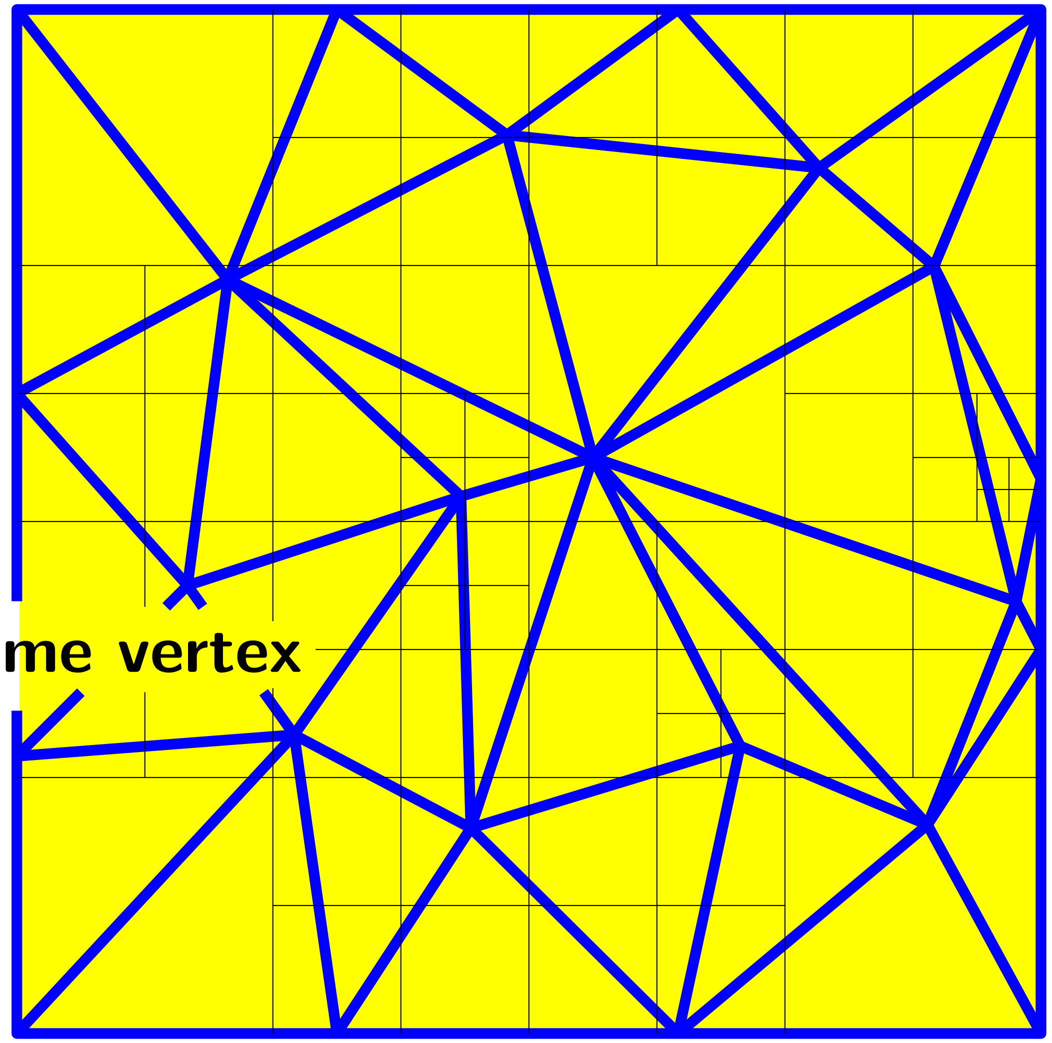

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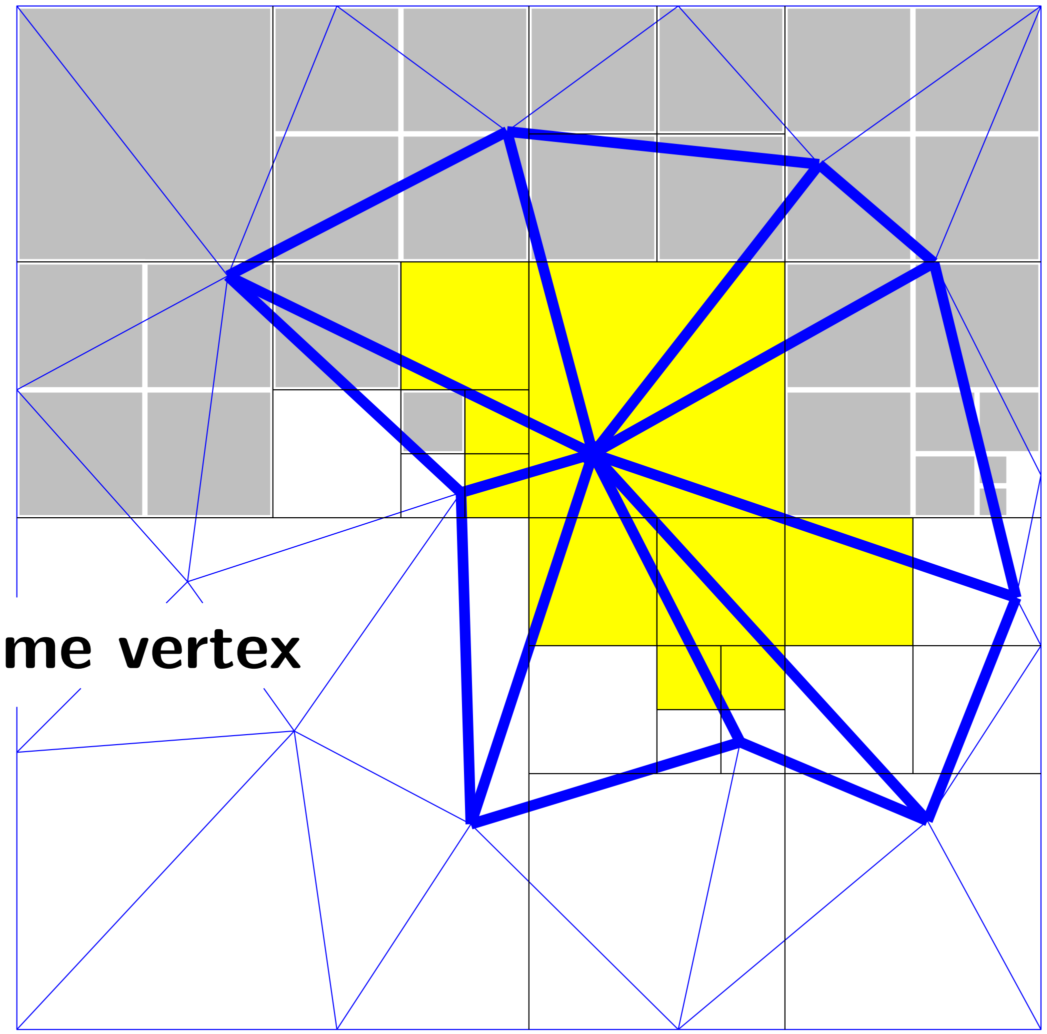
How to get that quadtree in Z-order (for triangulations of unit square)



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To prove for input of n triangles:

- together cells form subdivision of unit square;
- $O(1)$ triangles per cell;
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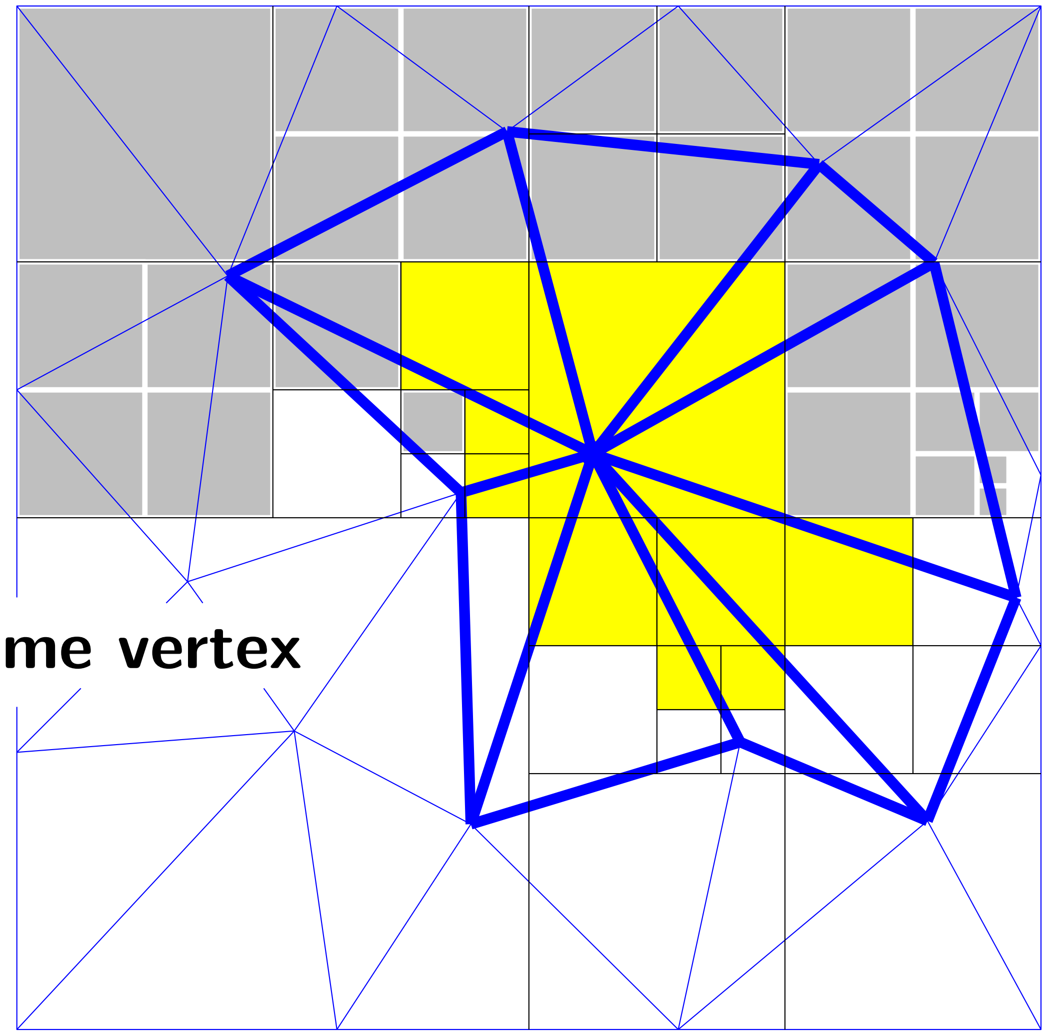
- algorithm runs in $O(sort(n))$ I/O's

How to get that quadtree in Z-order (for triangulations of unit square)



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- together cells form subdivision of unit square;
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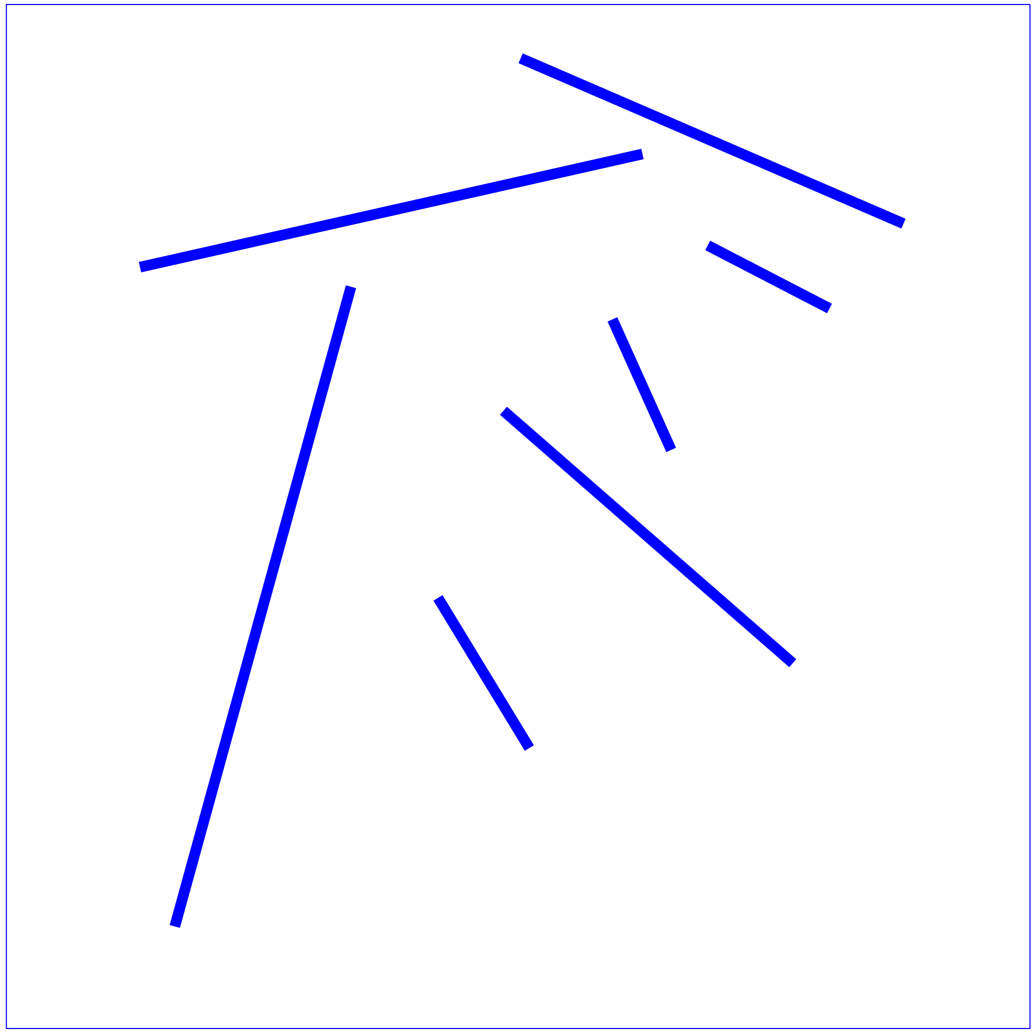
Works if triangles are *fat*:
minimum angle $>$
positive constant independent of n

How to get that quadtree in Z-order (for line segments in unit square)

Input: file with for each line segment its endpoints.



Algorithm:

1. Sort bounding box vertices of line segments into list $L = \{L_1, ..., L_m\}$ in Z-order
2. For $i \leftarrow 1$ to m:
   - find smallest cell Q that contains L_i and L_{i+1};
   - output cell boundaries of Q and its subquadrants
3. Sort cell boundaries in Z-order (removing duplicates)
4. Put line segments in cells

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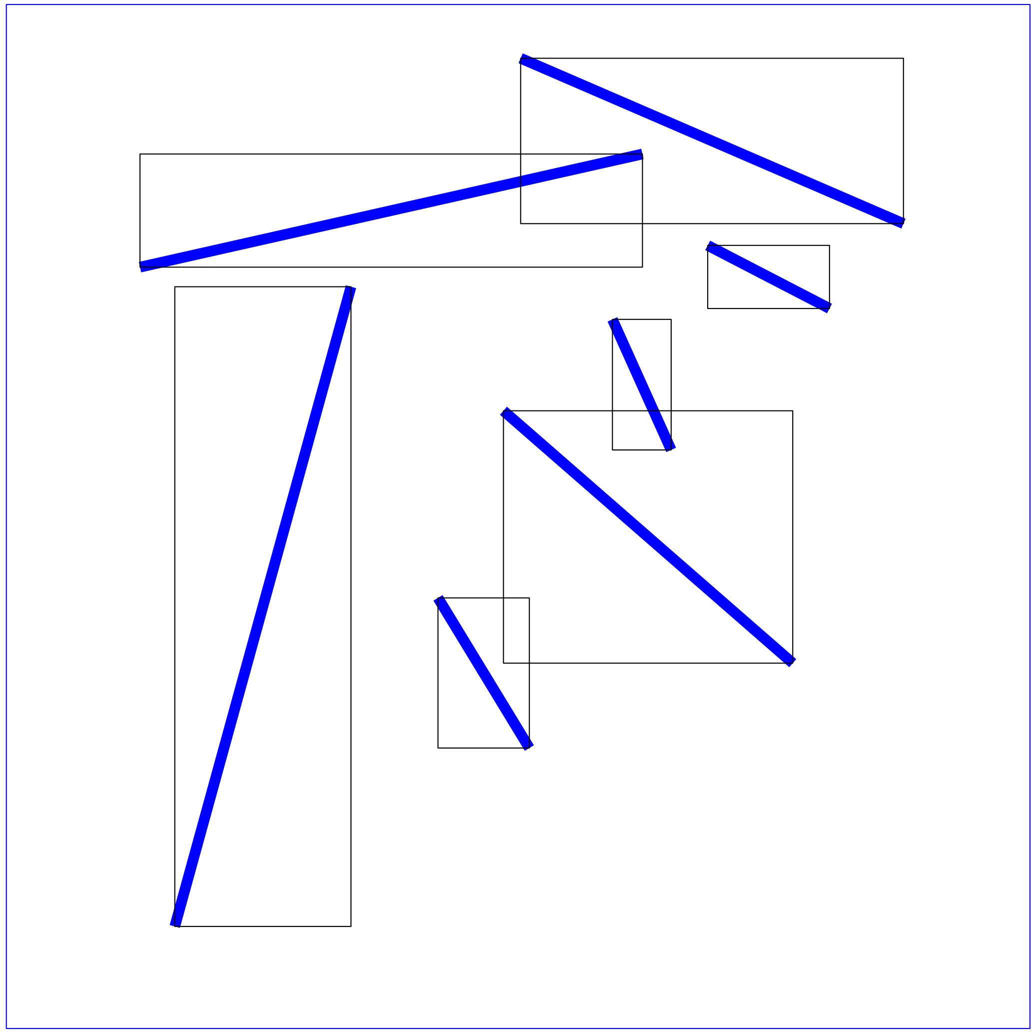

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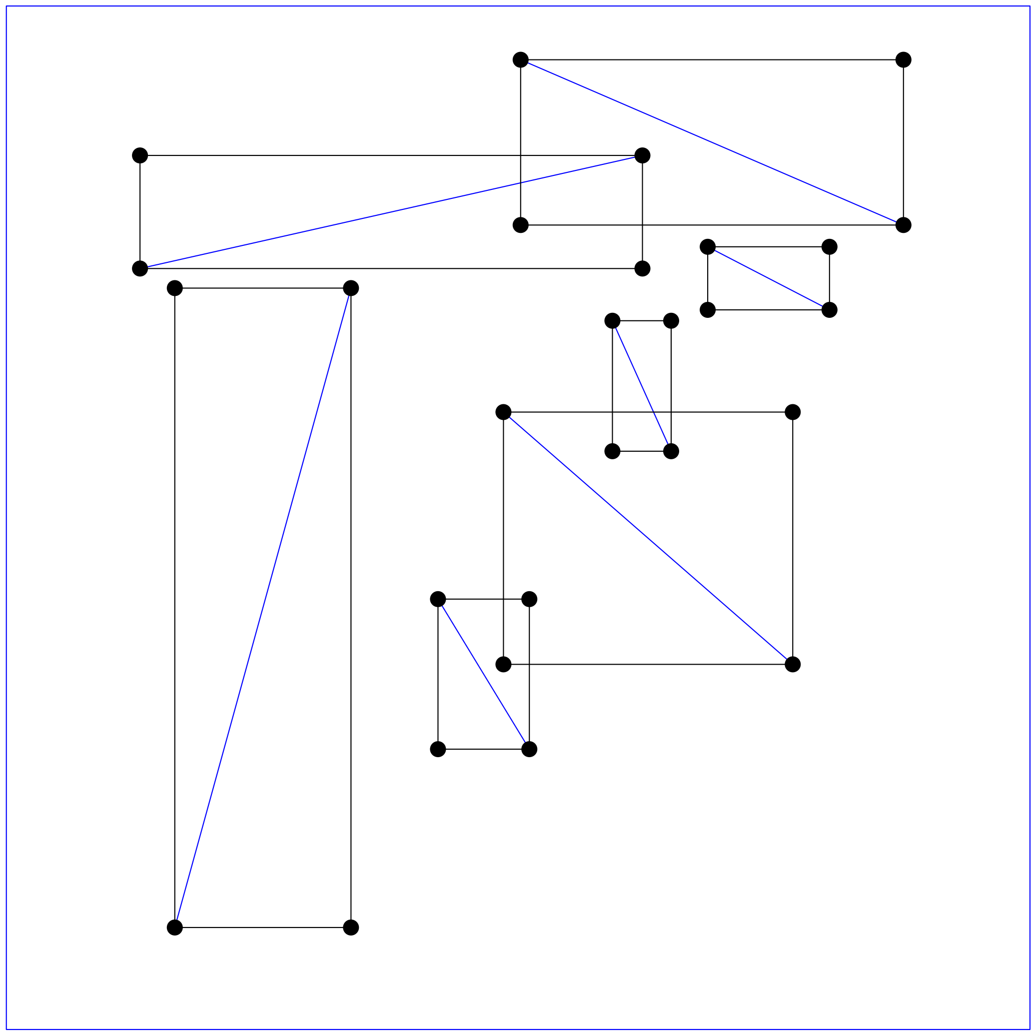

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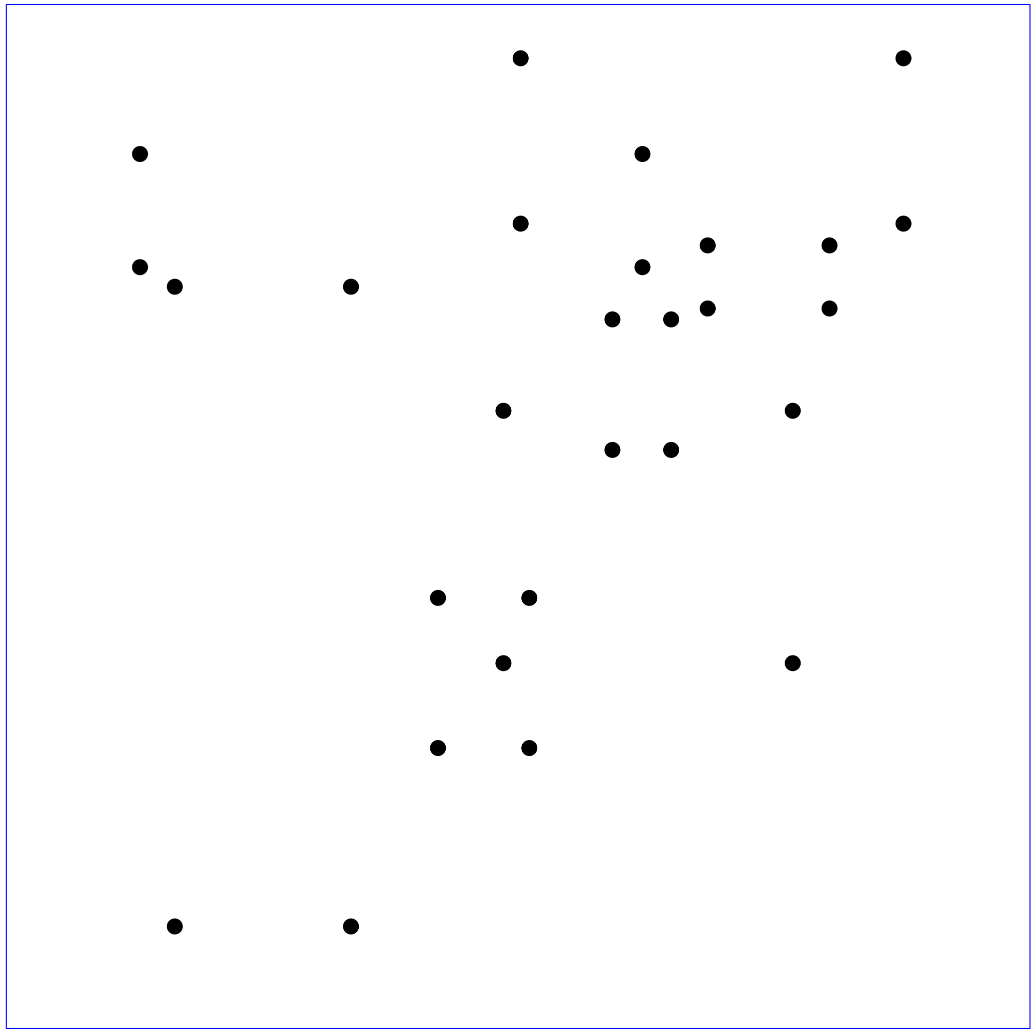

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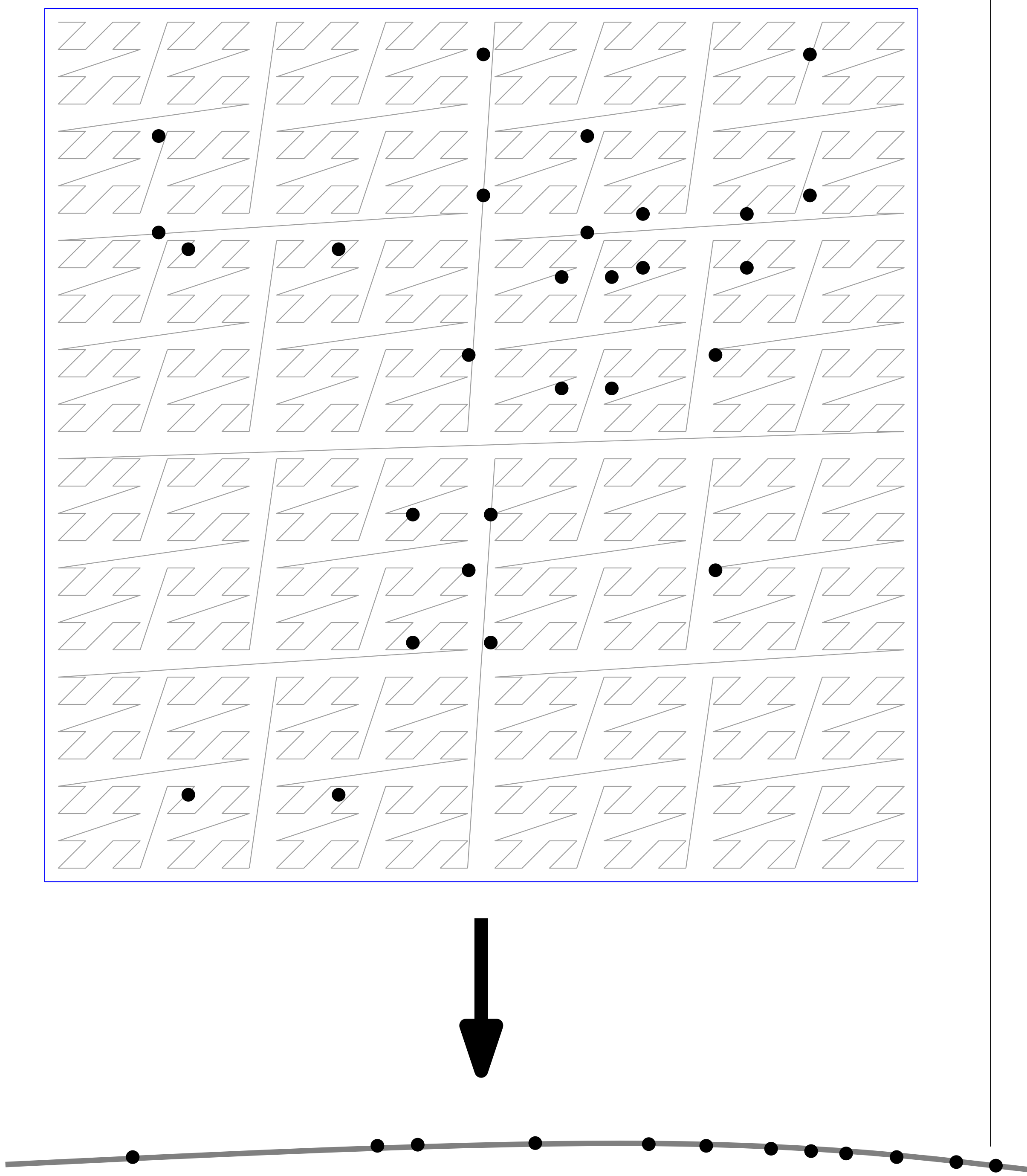

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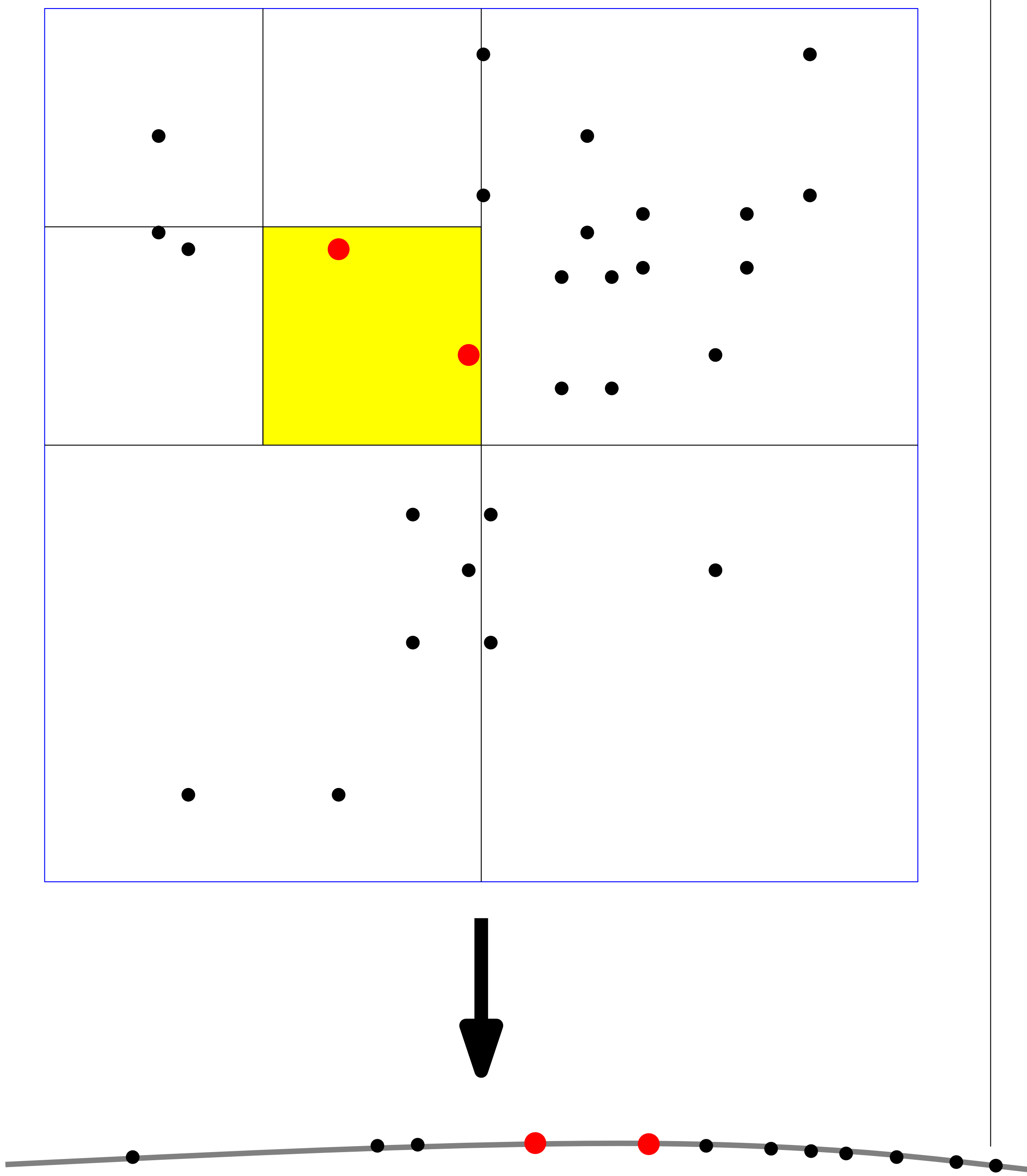
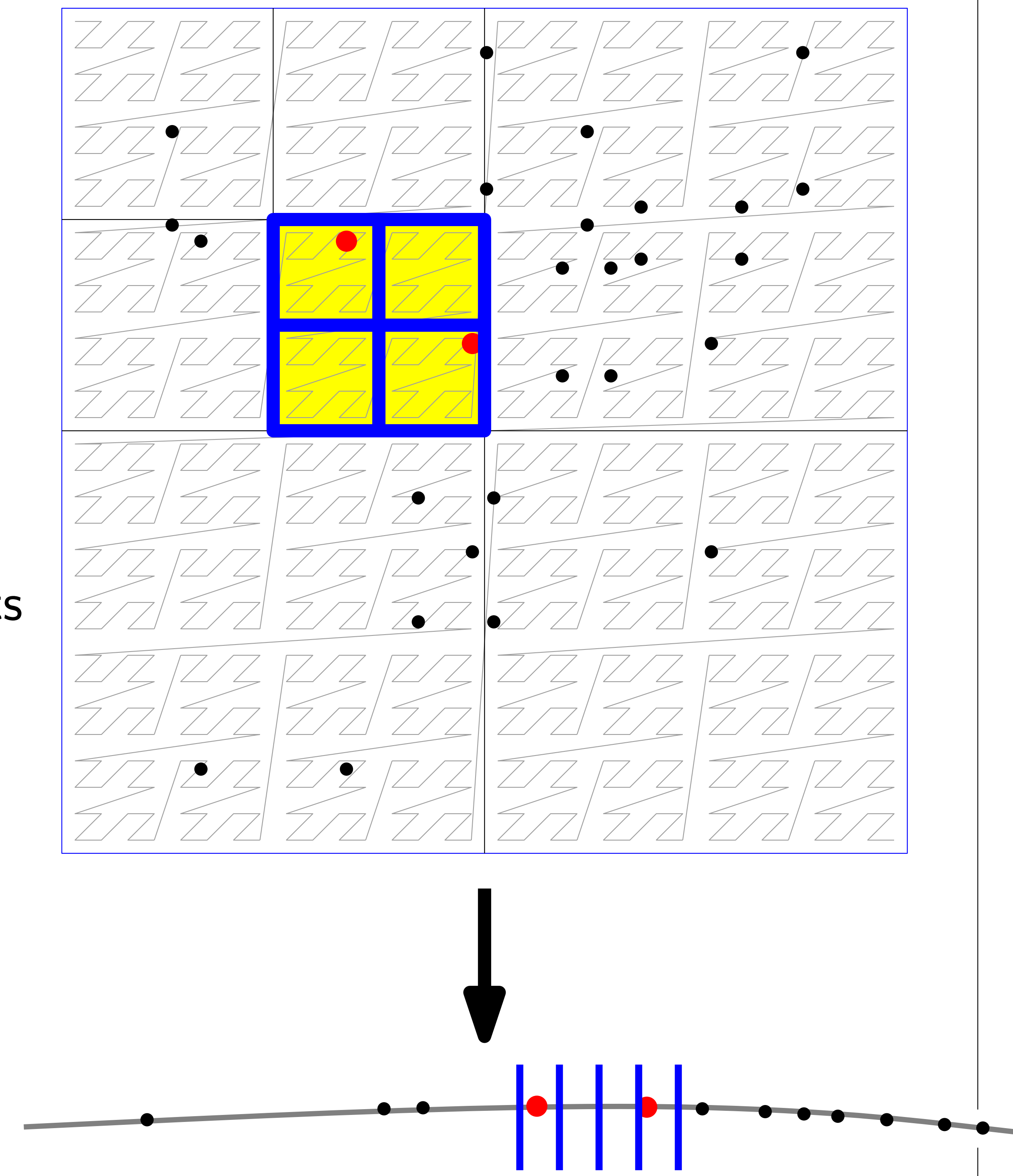


How to get that quadtree in Z-order (for line segments in unit square)

Input: file with for each line segment its endpoints.

Algorithm:

1. Sort bounding box vertices of line segments into list $L = \{L_1, ..., L_m\}$ in Z-order
2. For $i \leftarrow 1$ to m:
   - find smallest cell Q that contains L_i and L_{i+1};
   - output cell boundaries of Q and its subquadrants



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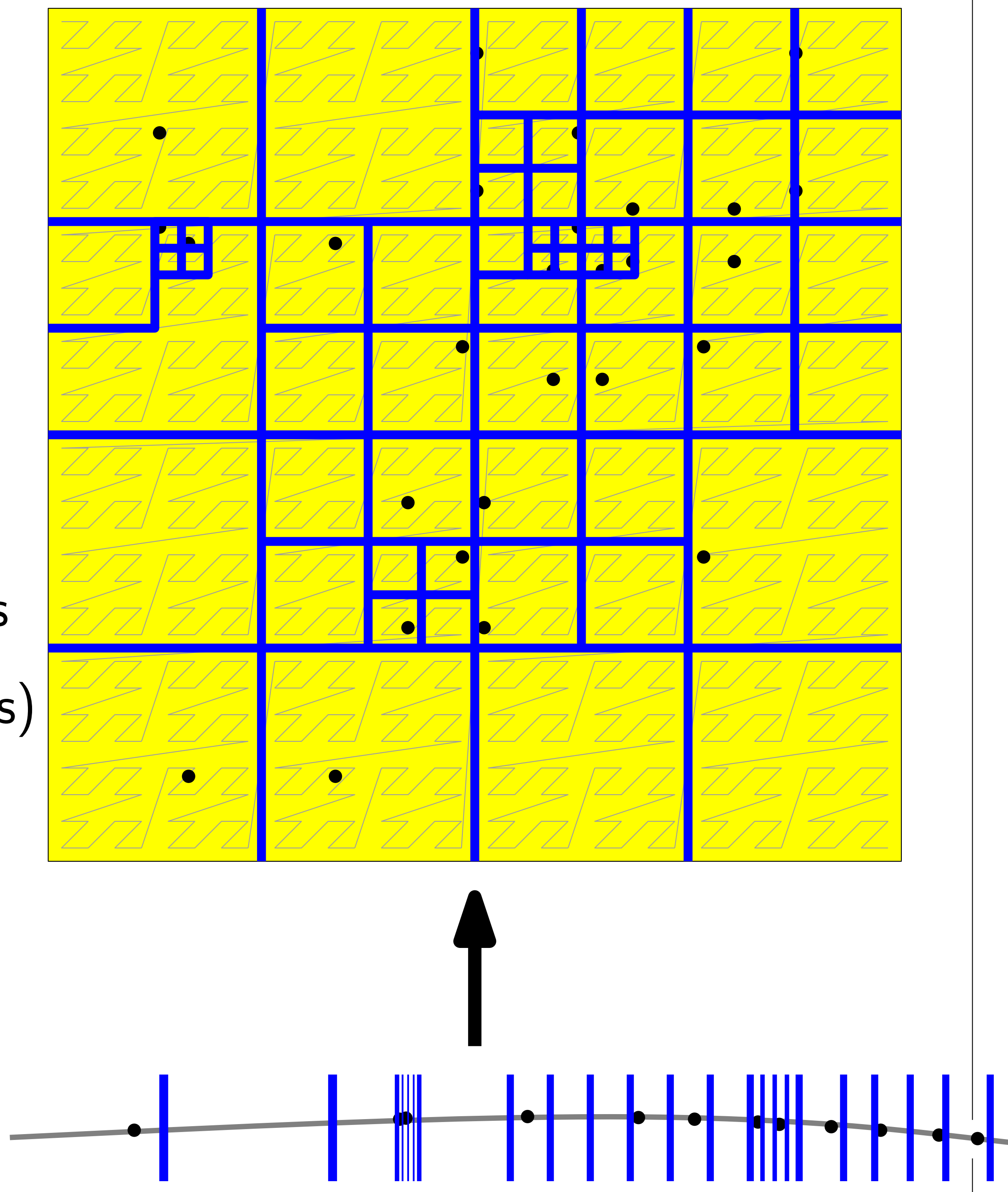

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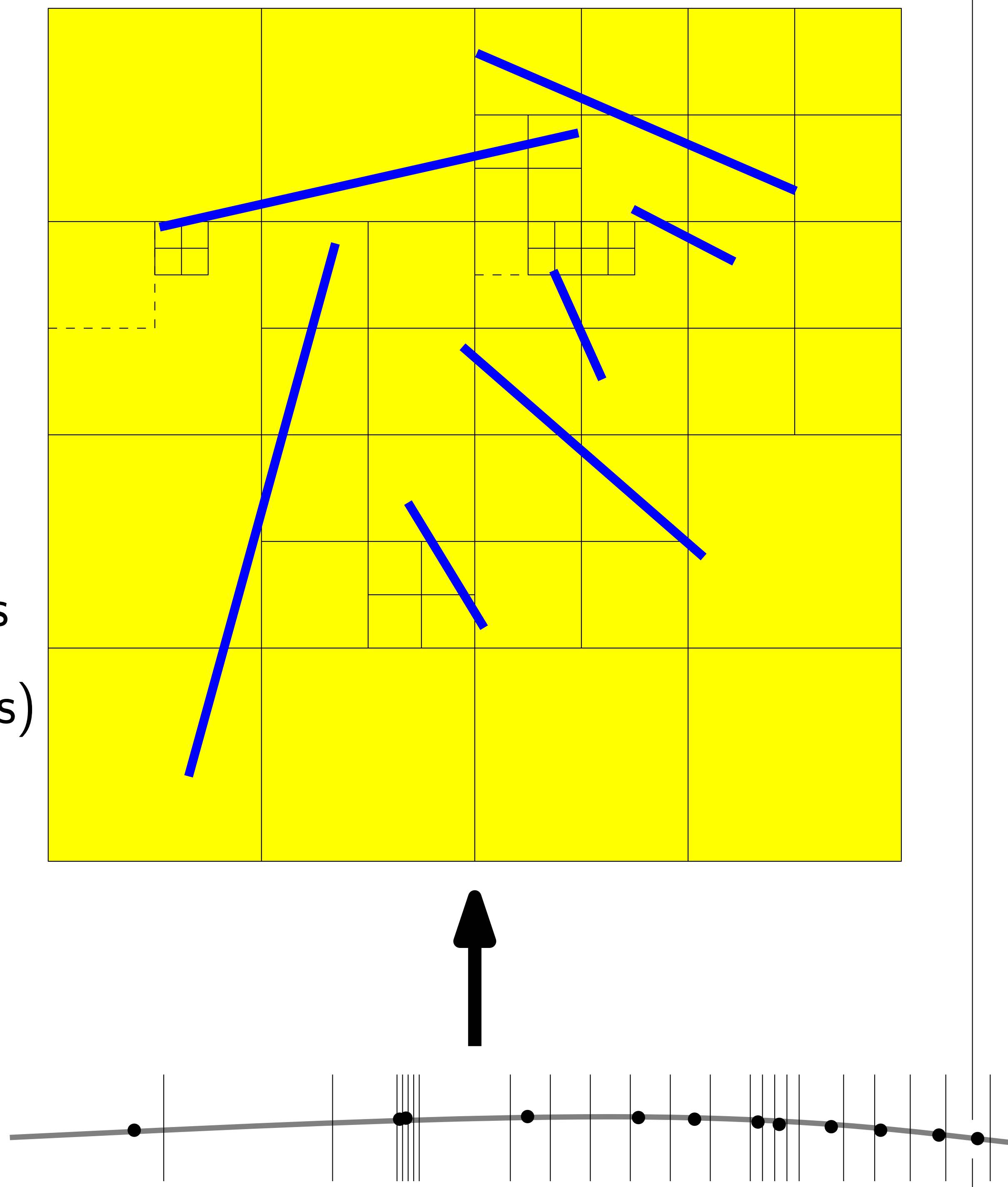

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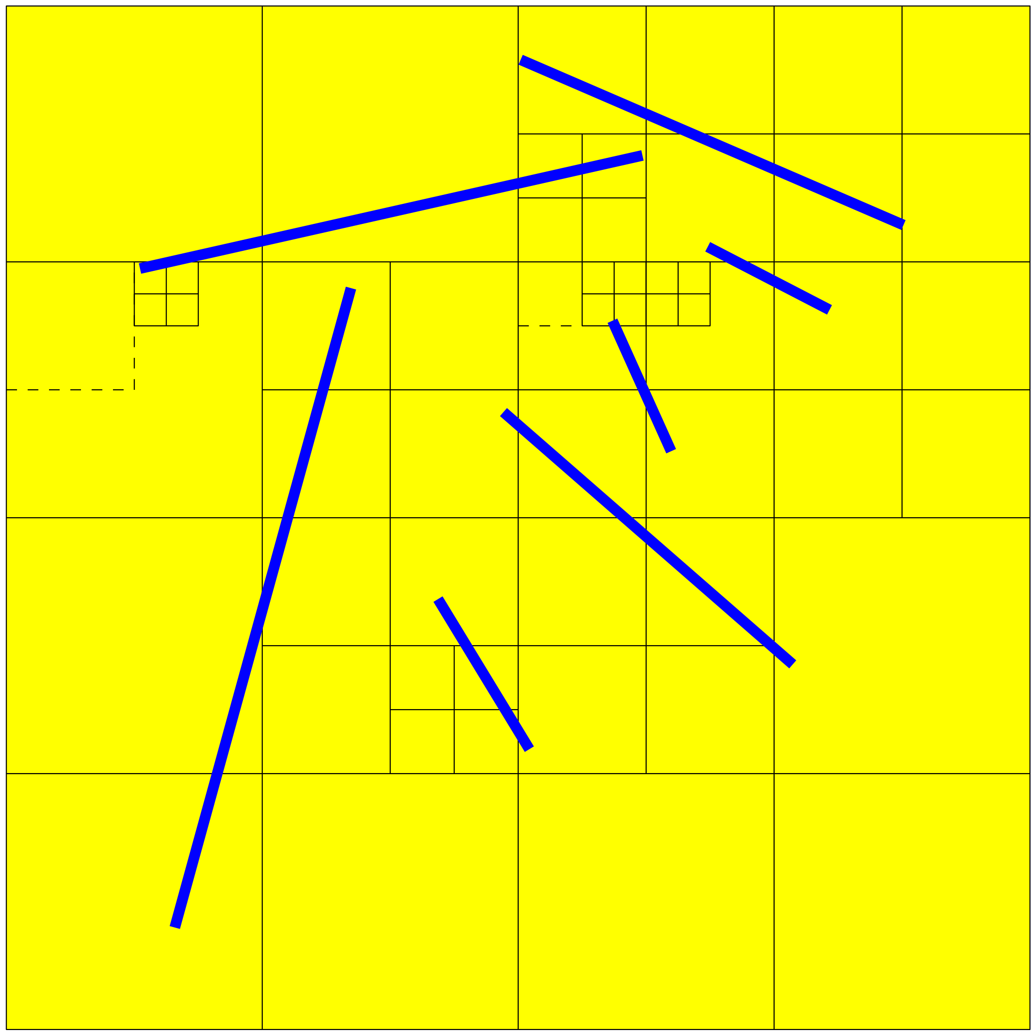

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To prove for input of n line segments:

- together cell boundaries form quadtree subdivision of unit square;
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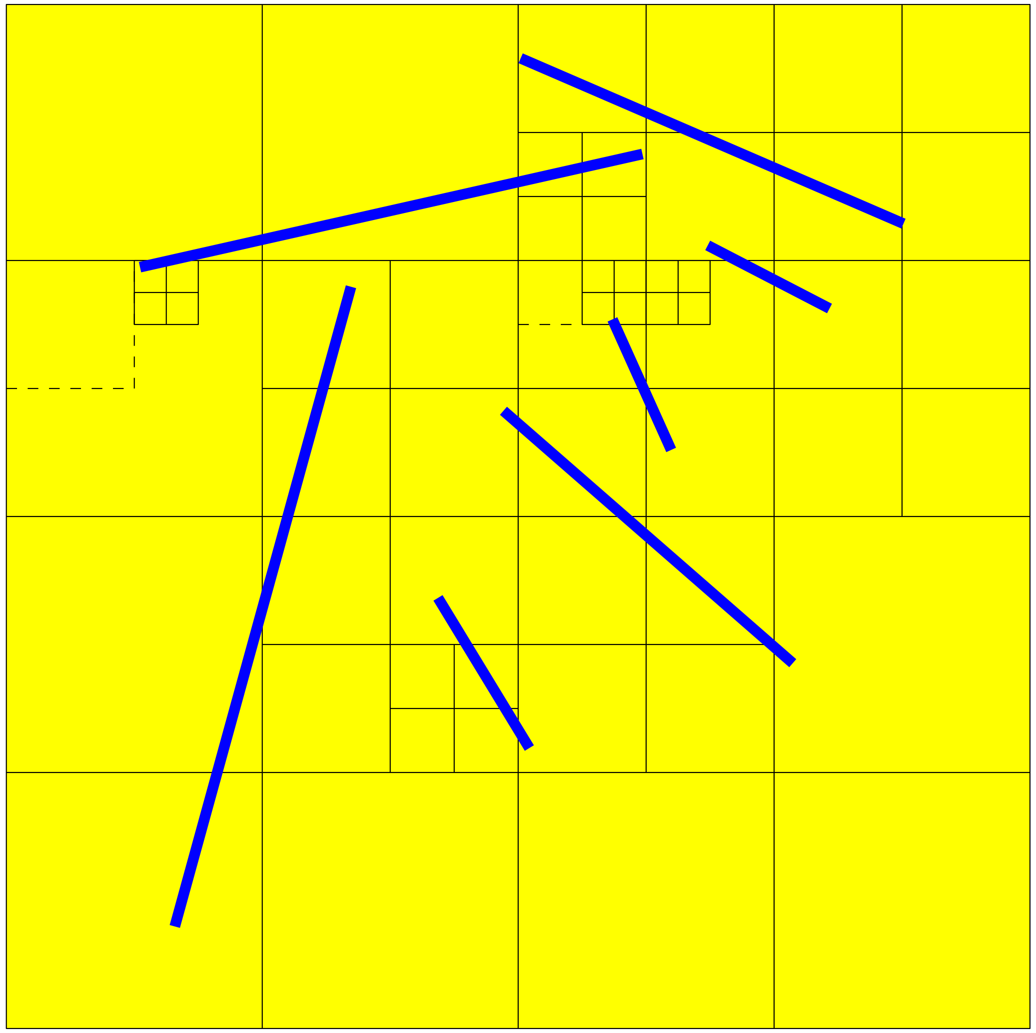

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To prove for input of n line segments:

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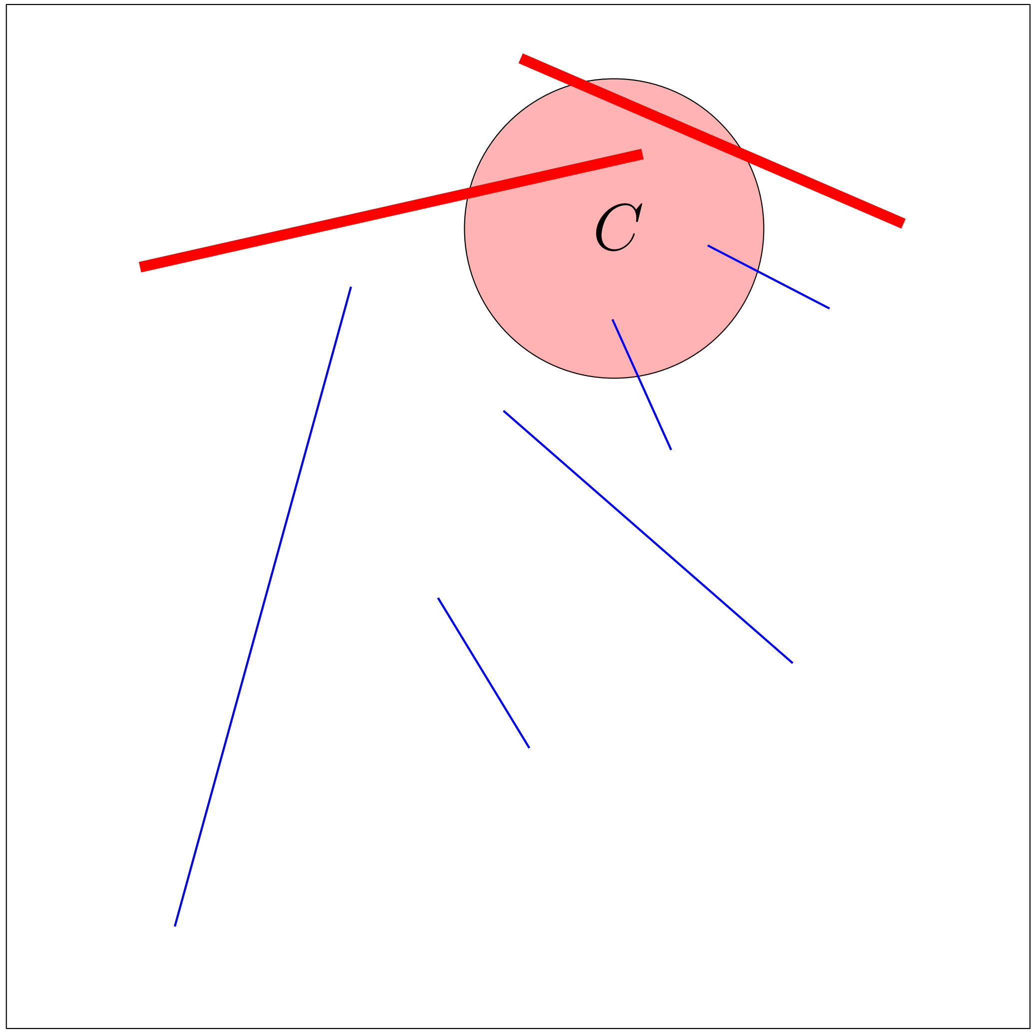

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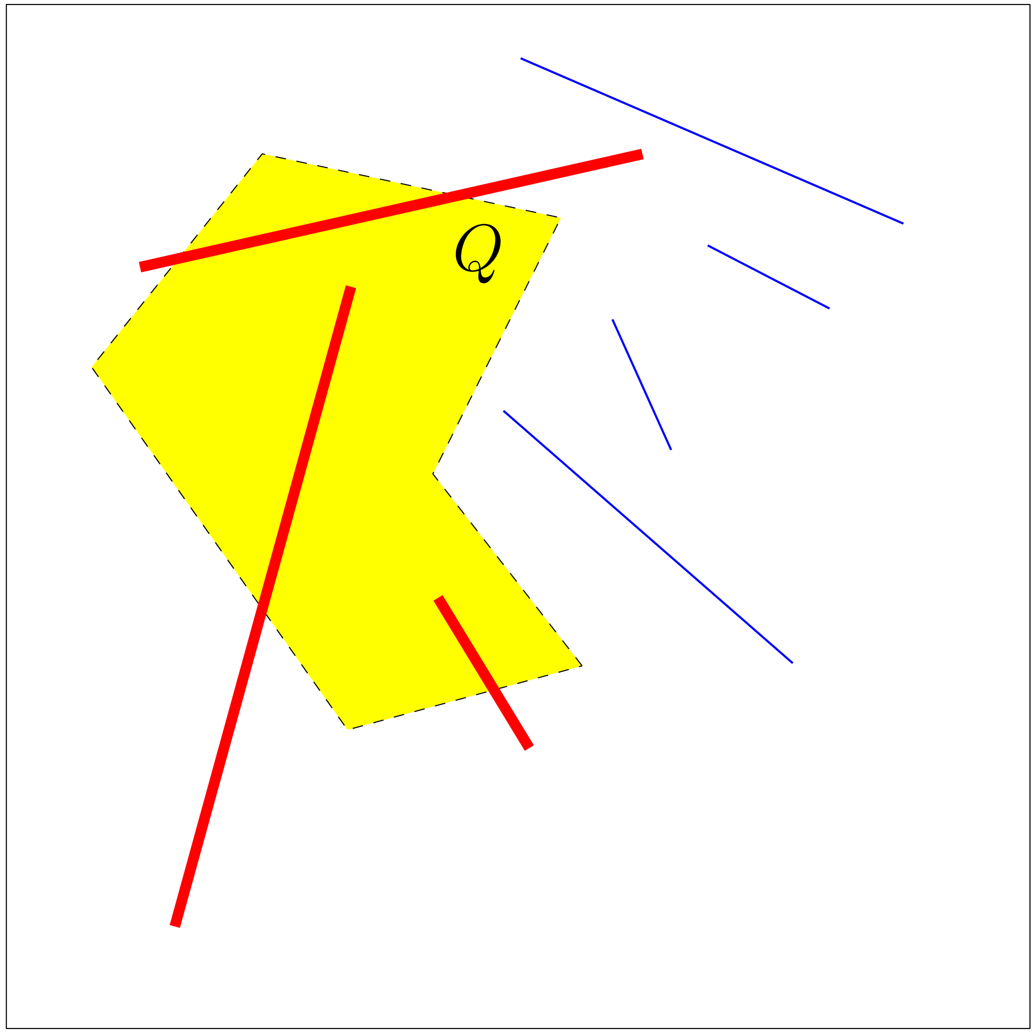
Works if line segments have *low density*:
for every circle C of diam d,
#line segments longer than d that intersect C
is at most a constant independent of n

Range queries

Report all line segments intersecting a query range Q of constant complexity



Report all line segments intersecting
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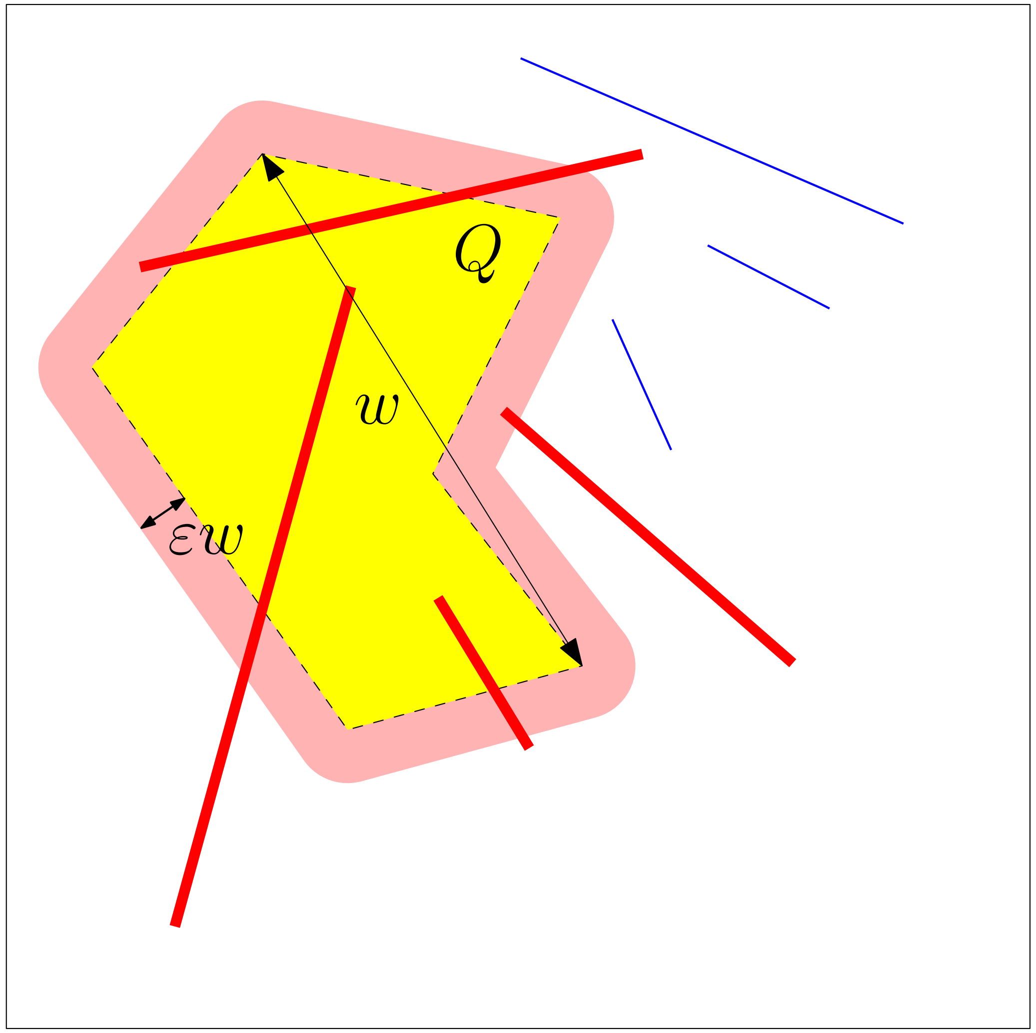
w = diameter of Q

k_{ε} = number of segments at distance $< \varepsilon w$ from Q



Range queries

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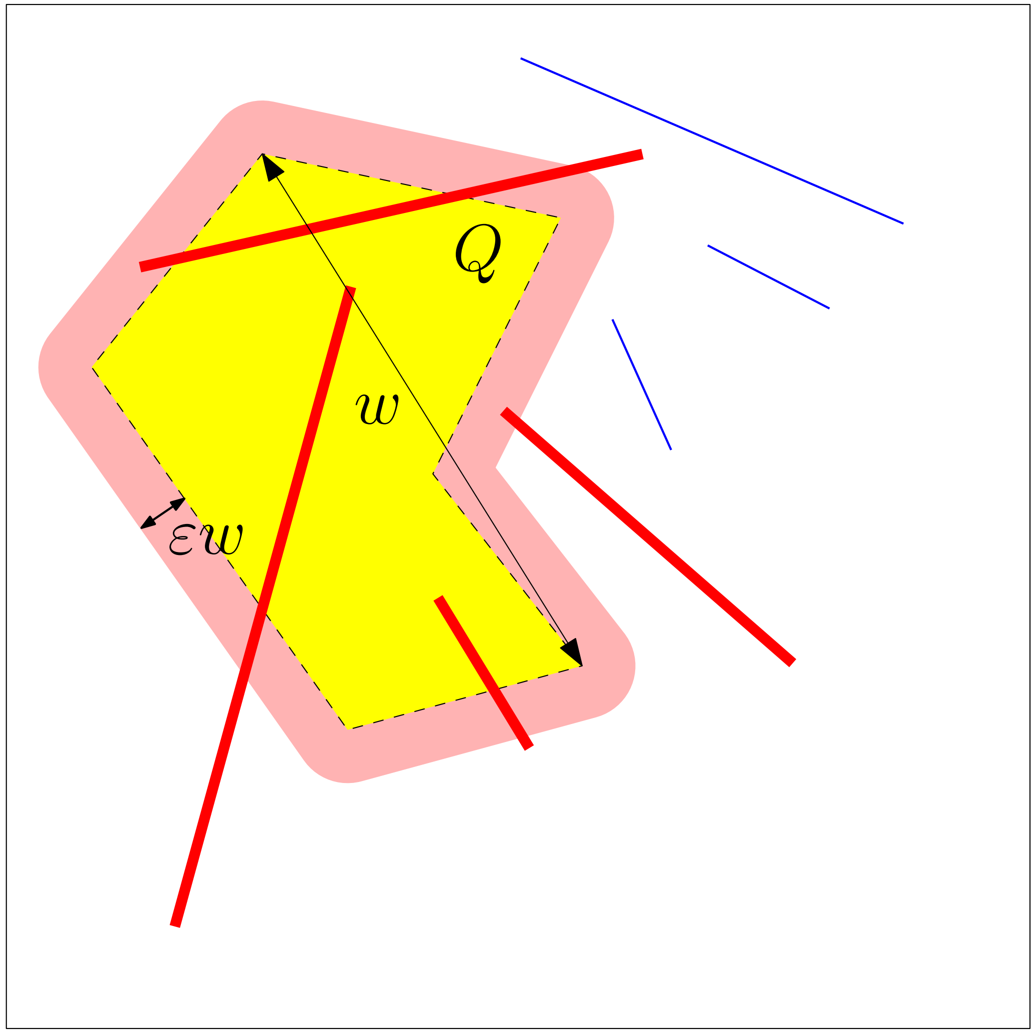
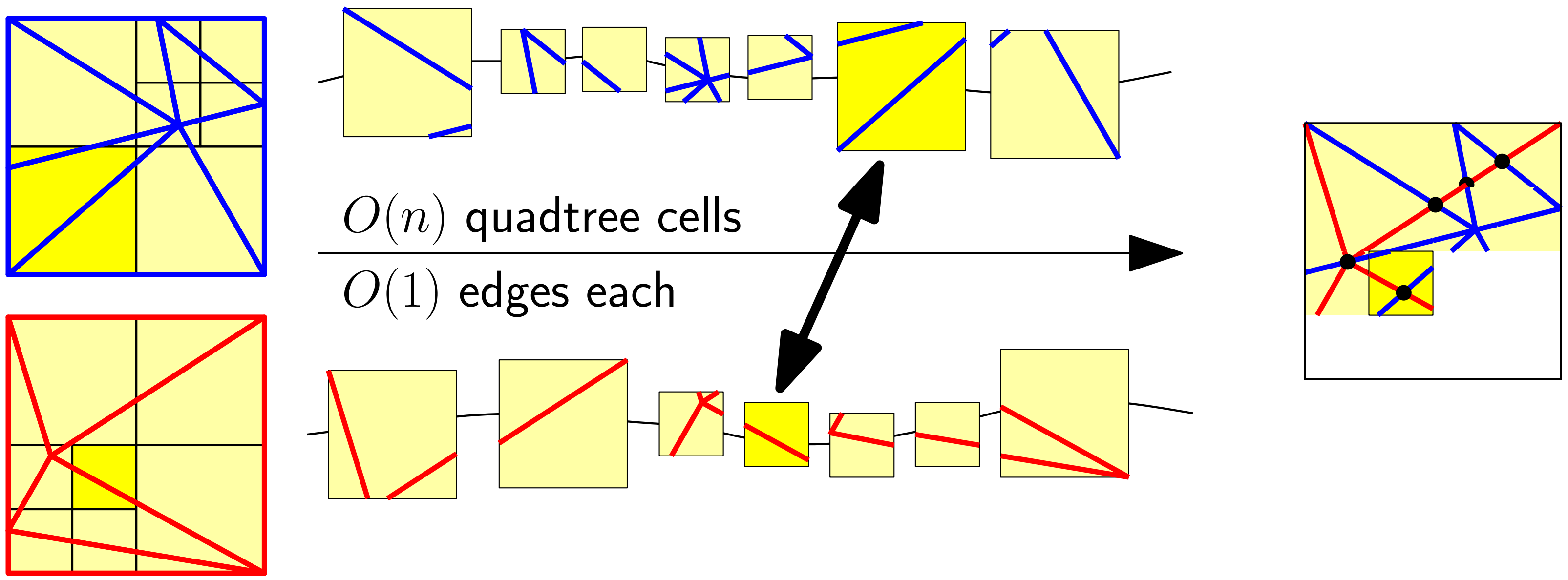
Results:

- for fat triangulations:
  range queries in $O(\frac{1}{\varepsilon}(\log_B n) + scan(k_\varepsilon))$ I/O's
- for low-density line segments:
  (after refining the data structure in $O(sort(n))$ I/O's)
  same bound.



I/O-Efficient Map Overlay and Point Location in Low-Density Subdivisions

Mark de Berg   Herman Haverkort   Shripad Thite   Laura Toma



n = input size; M = main memory size; B = disk block size; $scan(n) < sort(n) << n$

For low-density triangulations / sets of line segments*, there is a data structure that supports:

- map overlay in $O(scan(n))$ I/O's;
- point location in $O(\log_B n)$ I/O's;
- range queries in $O(\frac{1}{\varepsilon}(\log_B n) + scan(k_\varepsilon))$ I/O's.
- (triangulations only) updates in $O(\log_B n)$ I/O's;

The data structures are built with $O(sort(n))$ I/Os.

*) for any circle C, number of intersecting segments bigger than $\mathrm{diam}(C)$ is at most a constant

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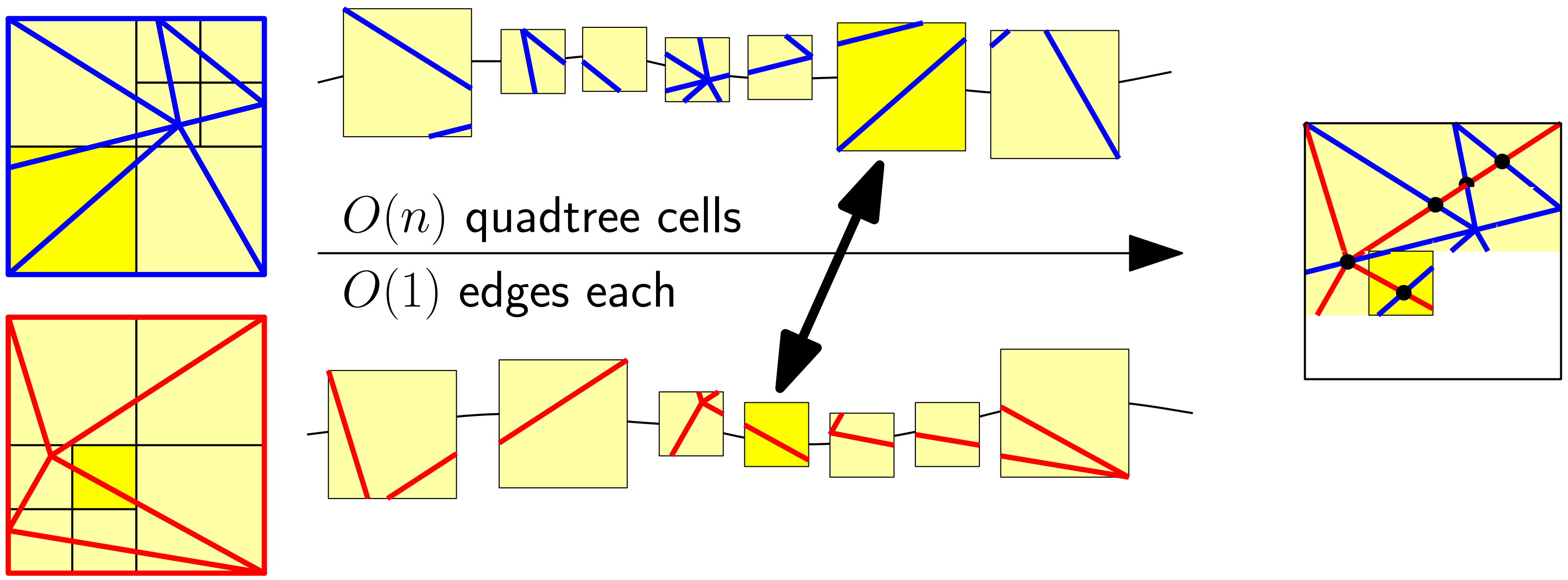

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That's all folks

*) for any circle C, number of intersecting segments bigger than $\mathrm{diam}(C)$ is at most a constant