Terracost: A Versatile and Scalable Approach to Computing Least-Cost-Path Surfaces for Massive Grid-Based Terrains

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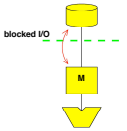
Least-cost path surfaces

- Problem
  - Input: A cost surface of a grid terrain and a set of sources
  - Output: A least-cost path surface: each point represents the shortest distance to a source
- Applications
  - Spread of fires from different sources
  - Distance from streams or roads
  - Cost of building pipelines or roads

I/O-Efficient Algorithms

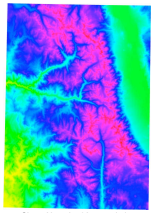
- Input data (grid) stored on disk
- I/O-model [Agarwal and Vitter, 88]
  - N = size of grid
  - M = size of main memory
  - B = size of disk block
  - I/O-operation (I/O): Reading/Writing one block of data from/to disk
- I/O efficiency
  - Number of I/Os performed by the algorithm
- Basic I/O bounds
  - Scanning: Scan(*N*) = *θ*(*N*/*B*) I/Os
  - Sorting: Sort(*N*) = *θ*(*N*/*B* $\log_{M/B}$ *N*/*B*) I/Os
  - In practice M and B are big:
    - Scan(N) < Sort(N) << N I/Os

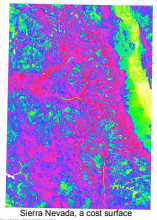


Terracost

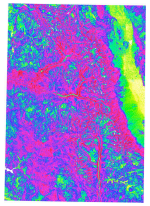
- Scalable approach to computing least-cost path surfaces on massive terrains
  - Based on optimal I/O-efficient algorithm: O(Sort(N)) I/Os
- Experimental analysis on real-life data
  - Can handle bigger grids
  - Can handle more sources
- Versatile: Interpolate between versions optimized for I/O or CPU
- Parallelization on a cluster

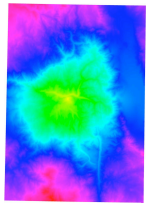
Grid terrains


Sierra Nevada, 30m resolution


Sierra Nevada, a cost surface

Least-Cost Surface with one source


Cost surface


Least-cost path surface (1 source)

Outline

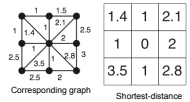
- Background
  - Shortest paths
  - Related Work
  - Shortest paths in the I/O-Model
- Terracost
  - Algorithm
  - Experimental analysis
  - Cluster implementation
- Conclusions

Shortest Paths

- Least-cost path surfaces correspond to computing shortest paths
- Shortest paths
  - Ubiquitous graph problem
  - Variations
    - SSSP: Single source shortest path
    - MSSP: Multiple source shortest path
- Grid terrains --> graphs

1	1	2
1	1	3
4	1	3

Cost grid

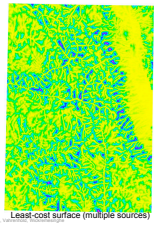

Corresponding graph

1.4	1	2.1
1	0	2
3.5	1	2.8

Shortest-distance from center point

Least-cost surface with Multiple Sources


Multiple sources


Least-cost surface (multiple sources)

Least-cost path surfaces on massive terrains

- Why massive terrains?
  - Large amounts of data are becoming available
    - NASA SRTM project: 30m resolution over the entire globe (~10TB)
    - LIDAR data: sub-meter resolution
- Traditional algorithms designed in RAM model don't scale
  - Buy more RAM?
    - Data grows faster than memory
  - Data does not fit in memory, sits on disk
  - Random I/O + Virtual memory => swapping

  => I/O-bottleneck

Related Work

- Dijkstra's Algorithm
  - Best known for SSSP/MSSP on general graphs, non-negative weights
- Recent variations on the SP algorithm
  - Goldberg et al SODA 200, WAE 2005
  - Kohler, Mohring, Schilling WEA 2005
  - Gutman WEA 2004
  - Lauther 2004
- Different setting
  - Point-to-point SP
    - E.g. Route planning, navigation systems
  - Exploit geometric characteristics of graph to narrow down search space
    - Route planning graphs
  - Use RAM model
  - When dealing with massive graphs ==> I/O bottleneck

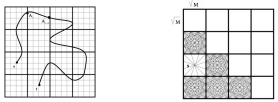
Dijkstra's Algorithm

1: Insert sources in a priority queue(PQ)
2: While PQ is not empty
3: DeleteMin vertex *u* with the least cost from PQ
4: Relax all edges incident to u

- In external memory
  - Dijkstra's algorithm requires 3 main data structures:
    1: Priority queue (can be implemented I/O-efficiently)
    2: Grid of costs (size = N >> M)
    3: Grid of current shortest path (size = N >> M)
  - Each time we DeleteMin from PQ, for every adjacent edge (u,v) we must do a lookup in both grids.
    - To check whether v can be relaxed
  - These lookups can cost O(1) I/Os each in the worst case

  ==> Total O(*N*) I/Os

I/O-Efficient SSSP on Grids [ATV'01]



0: Divide grid G into subgrids(G) of size O(M)

1: Construct a substitute graph S on the boundary vertices
- Replace each subgrid with a complete graph on its boundary
- For any u, v on the boundary of G_s the weight of edge (u,v) in S is $SP_{Gs}(u,v)$

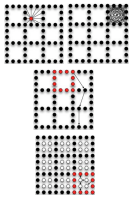
Lemma: S has $O(N/\sqrt{M})$ vertices, O(N) edges and it preserves the SP in G between any two boundary vertices u, v.

2: Solve SP in S
- Gives SP for all boundary vertices in G

3: Compute SP to vertices inside subgrids

Terracost

- **Step 1: (intra-tile Dijkstra)**
  - Partition into tiles of size R
  - Compute an edge-list representation of substitute graph S
    - Dijkstra from each boundary to tile boundaries
    - Dijkstra from sources to tile boundaries
- **Step 2:**
  - Sort boundary-to-boundary stream
- **Step 3: (inter-tile Dijkstra)**
  - Dijkstra on S
- **Step 4: (final Dijkstra)**
  - MSSP for each tile



Conclusions and Future Work

Key Points
- Dijkstra's algorithm is I/O-inefficient on large data sets
- Terracost restructures the input grid to run I/O-efficiently
  - But we can't ignore CPU-complexity completely
- I/O-bottleneck increases with number of sources for MSSP
- Tiling inTerracost allows for parallelization

Future Work
- Determine the optimal tile size analytically
- Find I/O-efficient SSSP/MSSP w/o increase of CPU-efficiency

Thank you.

Experimental Analysis

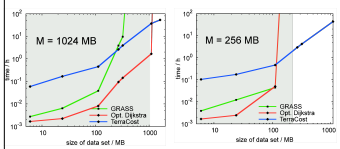
Dataset	Grid Size (million elements)	MB (Grid Only)
Kaweah	1.6	6
Puerto Rico	5.9	24
Hawaii	28.2	112
Sierra Nevada	9.5	38
Cumberlands	67	268
Lower New England	77.8	312
Midwest USA	280	1100

Experimental Platform
- Apple Power Macintosh G5
- Dual 2.5 GHz processors
- 512 KB L2 cache
- 1 GB RAM

Compare Terracost with r.cost in GRASS
- r.cost has same functionality
- GRASS users have complained it is very slow for large terrains

Experimental Analysis

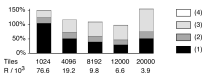
- GRASS: r.cost
- Opt Dijkstra: internal memory version of Terracost (num tiles = 1)
- Terracost: I/O-efficient version of Terracost



CPU-I/O Tradeoff

R = tile size
- I/O-complexity = O(N / √R + sort(N))
  - Dominated by Step 3 (inter-tile Dijkstra)
- CPU-complexity = O(N √R log R)
  - Dominated by Step 1 (intra-tile Dijkstra)

- So, to optimize I/Os, we want a large R.
- But, to optimize CPU, we want a small R.
- Optimal performance: balance I/O-CPU



Lower NE, Cost = Slope, 1GB RAM, Single source

Terracost on Clusters

- Terracost lends itself to parallelization
- We parallelized the most CPU-intensive part
  - Computing the substitute graph (Step 1)
- Hgrid
  - Cluster management tool
  - Clients submit requests (run jobs, query status); agents get jobs and run them
- Near-linear speedup