CSCI 189 Assignment 1 Problem Set

This assignment povides an opportunity to use propositional logic for representing arguments and exploring their validity.

Summary of Concepts in Section 1.1-1.2

Logic

A *wff*, or *proposition* is any statement that can be *true* or *false* (T or F). Wff's are classified as:

Simple variables $P, Q, R, ...$

Conjunctions, of the form $P \wedge Q$, which is T when both P and Q are T.

Disjunctions, of the form $P \vee Q$, which is T when either P or Q (or both) is T.

Negations of the form P' which is T when P is F.

Implications, of the form $P \rightarrow Q$, which is T unless P is T and Q is F, and

Equivalences, of the form $P \leftrightarrow Q$, which is T when both P and Q are T or F simultaneously.

A *tautology* is a wff that is always *true*. A *contradiction* is a wff that is always *false*. An *equivalence* occurs when two wff's have the same truth table. A *truth table* is a list containing the value of a wff for every different setting (T or F) for each if its variables. Below is a truth table for the wff $A \wedge (B \rightarrow C)'$, which is neither a tautology nor a contradiction.

A	B	C	$A \wedge (B \rightarrow C)'$
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

Logic can be used to check the validity of an argument. An *argument* is a wff of the form:

$P_1 \wedge P_2 \wedge \cdots \wedge P_n \rightarrow Q$

where the P's are the *hypotheses* and Q is the *conclusion*. An argument is *valid* if its wff is a tautology.

Proof

A *proof* of an argument is a sequence of wffs in which each wff is either a hypothesis or else it can be derived from earlier wffs in the sequence using a *derivation rule*; the last wff in the sequence must be the argument's conclusion. Each wff in the sequence is accompanied by a "justification," which is a brief notation of what derivation rule and what prior steps were used to arrive at this wff.

Derivation rules come in two flavors; *equivalence rules* and *inference rules*, whose meanings are summarized in the table below:

Equivalence Rule	Meaning	Inference Rule	Meaning
Commutativity (comm)	$P \wedge Q \leftrightarrow Q \wedge P$	Modus Ponens (mp)	$P, P \rightarrow Q \mid -Q$
Associativity (ass)	$(P \wedge Q) \wedge R \leftrightarrow P \wedge (Q \wedge R)$	Modus Tollens (mt)	$P \rightarrow Q, Q' \mid -P'$
	$(P \vee Q) \vee R \leftrightarrow P \vee (Q \vee R)$		
DeMorgan's laws	$(P \wedge Q)' \leftrightarrow P' \vee Q'$	Conjunction (con)	$P, Q \mid -P \wedge Q$
	$(P \vee Q)' \leftrightarrow P' \wedge Q'$		
Implication (imp)	$P \rightarrow Q \leftrightarrow P' \vee Q$	Simplification (sim)	$P \wedge Q \mid -P$
Double negation (dn)	$(P')' \leftrightarrow P$	Addition (add)	$P \mid -P \vee Q$

For any equivalence rule, if the expression on the left of $\leftrightarrow$ appears in a proof, it can be replaced in a later step by the expression on the right (and vice-versa).

For any inference rule, if the expression(s) on the left of $\mid -$ appear in a proof, they can be replaced in a later step by the expression on the right (but *not* vice versa). A careful discussion of proof-building appears on pages 25-29 of your text. Below is a carefully developed proof of the proposition given at the beginning of the chapter (page 1), using a 4-step methodology. The first three steps are easy; the fourth is challenging.

First, assign variables to statements in the proposition:

G = "my client is guilty"
D = "the knife was in the drawer"
J = "Jason Pritchard saw the knife"
O = "the knife was there on October 10"
H = "the hammer was in the barn"

Second, encode the individual statements into wff's, using your variables:

a.	$G \rightarrow D$	"If my client is guilty, then the knife was in the drawer."
b.	$D' \vee J$	"Either the knife was not in the drawer or Jason Pritchard saw the knife."
c.	$O' \rightarrow J'$	"If the knife was not there on Oct 10, then Jason Pritchard didn't see the knife."
d.	$O \rightarrow D \wedge H$	"If the knife was there on Oct 10, then the knife was in the drawer and the hammer was in t
e.	H'	"The hammer was not in the barn."
f.	G'	"Therefore, my client is innocent."

Notice that the basic form of of this argument is $a \wedge b \wedge c \wedge d \wedge e \rightarrow f$.

Third, list the first steps in the proof as hypotheses from the problem statement.

1.	$G \to D$	Hyp
2.	$D' \vee J$	Hyp
3.	$O' \to J'$	Hyp
4.	$O \to D \wedge H$	Hyp
5.	H'	Hyp

Fourth (this is the tricky part), develop a series of steps that leads to the conclusion, G', or "my client is innocent."

6.	$D' \vee H'$	5, add
7.	$(D \wedge H)'$	6, DeMorgan
8.	O'	4, 7, mt
9.	J'	3, 8, mp
10.	$D \to J$	2, imp
11.	D'	9, 10, mt
12.	G'	1, 11, mt

Problems to be handed in

You may choose to work in groups of 2 or 3 to complete this assignment. In that case, each group member should contribute a fair share of the work, and the group should turn in one set of answers (listing the names of group members at the top).

Section 1.1 (p 14) Complete Exercises 2, 5, 7, 8ace, 9ace, 10ace, 13, and 14ceg. Use Haskell, as described in the lab, to generate a truth table for each of the propositions in Exercise 14ceg. Check that these truth tables agree with the ones you developed by hand.

Section 1.2 (p 30) Exercises 2, 4, 11, 12, 17, 19, 24, 30, 34, 38, 40, 44.

Extra credit (optional - food for the soul!). Prove or disprove (i.e., prove the negation) any of the following arguments:

1. If you fail to complete all the assignments in this course, you will fail the course. If you fail the course, you cannot take more computer science courses. Therefore, if you take more computer science courses, you must have completed all the assignments in this course.
2. If we send an astronaut to Mars, then Mars is made of green cheese. If Mars is made of green cheese, then I am a monkey. Either nobody is on Mars or Mars is not made of green cheese. Therefore, I am a monkey.
3. The maid said she saw the butler in the living room. The living room adjoins the kitchen. The shot was fired in the kitchen and could be heard in all adjoining rooms. The butler, who had good hearing, said he did not hear the shot. Could he have heard it?