CSCI 189 Assignment 2 Problem Set

This assignment povides an opportunity to use predicate logic for representing arguments and exploring their validity. It also provides some hands-on experience with using the LogicDaemon proof checker.

Summary of Concepts in Section 1.3-1.4

Quantifiers and Predicates

The predicate $P(x)$ denotes a property for variable x in some *domain* of values. E.g., $x > 0$ is a predicate about some value in the domain of numbers. "x likes solving jigsaw puzzles." is a predicate about someone in the domain of people. Like propositions, a predicate has a truth value (T or F).

$\forall x P(x)$ means "every x in the domain has the property $P(x)$. E.g., "Every number is positive," or "Everyone in our class likes solving crossword puzzles."

$\exists x P(x)$ means "some (one or more) x in the domain has property $P(x)$. E.g., "Some numbers are positive," or "Some people like solving crossword puzzles."

Expressions that have $\forall x$ and/or $\exists x$ are called "quantified expressions", and the variable x in such expressions is called a "bound variable." Other variables in such expressions are called "free variables." E.g., in the expression $\forall x P(x, y)$ x is bound and y is free.

Here are some more examples:

$\forall x(book(x) \wedge wrote(Bertrand, x) \rightarrow read(Allen, x))$ "Allen read every book that Bertrand wrote."

$\exists x(book(x) \wedge borrowed(Chu, x, Allen))$ "Chu borrowed a book from Allen"

$\forall x \exists y(student(x) \rightarrow book(y) \wedge borrowed(x, y, Allen))$ "Every student borrowed a book from Allen."

A predicate is *valid* if it is true (T) for all interpretations (in all domains). A predicate is *invalid* if there is a domain (an interpretation) in which it is false (F).

Proof with Predicates

To extend the deductive proof paradigm to predicates, we can "instantiate" and "generalize" various forms, as suggested in the following equivalence and inference rules with quantifiers:

Equivalence Rule	Meaning	Inference Rule	Meaning
Negation (neg)	$(\forall x P(x))' \leftrightarrow \exists x P'(x)$	Universal Instantiation (ui)	$\forall x P(x) \mid\!- P(a)$
Negation (neg)	$(\exists x P(x))' \leftrightarrow \forall x P'(x)$	Existential Instantiation (ei)	$\exists x P(x) \mid\!- P(a)$
		Universal generalization (ug)	$P(x) \mid\!- \forall x P(x)$
		Existential generalization (eg)	$P(a) \mid\!- \exists x P(x)$

Here, "a" is some particular member of the domain of x. For example, if we know that read(Chu, Principia), existential generalization allows us to generalize to $\exists x read(x, Prinicipia)$.

Universal negation is illustrated by the equivalence of the following two statements:

"Not everyone read Principia."

"Someone did not read Principia."

Similarly, existential negation is illustrated by the following two equivalent statements:

"No one in the class likes crossword puzzles."

"Everyone in the class does not like crossword puzzles."

Here is an example proof with predicates that the following argument is valid. "Every student likes crossword puzzles. Some students like ice cream. Therefore, some students like ice cream and crossword puzzles." Let:

$S(x)$ = "x is a student"
$C(x)$ = "x likes crossword puzzles"
$I(x)$ = "x likes ice cream"

So the argument to be proved is: $\forall x(S(x) \rightarrow C(x)) \wedge \exists x(M(x) \wedge I(x)) \rightarrow \exists x(S(x) \wedge C(x) \wedge I(x))$. Here is a proof:

1.	$\forall x(S(x) \rightarrow C(x))$	Hyp
2.	$\exists x(S(x) \wedge I(x))$	Hyp
3.	$S(a) \wedge I(a)$	2, ei
4.	$S(a) \rightarrow C(a)$	1, ui
5.	$S(a)$	3, sim
6.	$C(a)$	4, 5, mp
7.	$S(a) \wedge C(a) \wedge I(a)$	3, 6, con
8.	$S(a) \wedge I(a) \wedge C(a)$	7, comm
9.	$\exists x(S(x) \wedge I(x) \wedge C(x))$	8, eg

Notice that this proof is identical with the proof of Example 35 (page 55), except that the domain is changed.

Problems to be handed in

Again, you are welcome to work in groups of 2 or 3 to complete this assignment. Each group member should contribute a fair share of the work, and the group should turn in one set of answers (listing the names of group members at the top).

Consider the two proofs you wrote for Assignment 1, Exercises 38 and 40 on page 32. Now encode each of these proofs into a form that the LogicDaemon proof checker will understand, and then use LogicDaemon to check your proofs. Turn in a snapshot of the LogicDaemon window that appeared when it checked each proof.

Section 1.3 (p 41) Exercises 2efg, 3ace, 4b, 6dfh, 8, 9adf, 15bd, 19bd.

Section 1.4 (p 57) Exercises 4, 7, 12, 16, 18, 26, 31, 34ace.

Extra credit (optional - more food for the soul). Prove or disprove either of the following arguments:

1. If God were able and willing to prevent evil, s/he would do so. If God were unable to prevent evil, s/he would be impotent; if s/he were unwilling to prevent evil, s/he would be malevolent. God does not prevent evil. If God exists, s/he is neither impotent nor malevolent. Therefore, God does not exist.
2. Each of the three presidential candidates is either a liar or a "truther" (that is, one who always tells the truth). At a recent public debate, candidate A claimed that candidates B and C were liars. B denied that s/he was a liar, but C claimed that B was a liar. Are there any truthers among these candidates? (Let A denote "A is a truther", and so A' denotes "A is a liar." So here you are trying to establish whether $A \vee B \vee C$ is valid.)